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Shirking, endogenous lay-off rates and the a-cyclicality of the real wage

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Abstract

The paper extends the Shapiro-Stiglitz (1987) efficiency wage model by endogenising the probability of worker displacement as a function of the change in the firm's employment. This creates counter-cyclical variation in the wage mark-up and thereby generates real wage persistence. A New Keynesian DSGE model equipped with this extension replicates the empirical facts of a very limited response of the real wage to the business cycle together with a lagged hump-shaped response of employment to output.

JEL classification: E24, E43

Keywords: Efficiency wages, shirking, real wage persistence, New Keynesian models

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1 Introduction

Real wages show little association with cyclical variations in labour productivity. As a result, the profit share turns strongly pro-cyclical. While nominal rigidities in price- and wage-setting partly explain these patterns, they are unlikely to fully account for the degree of real wage persistence found in the data (Dickens et al., 2006). Accordingly, there is a widespread view that real wage rigidities play an important role in the propagation of the business cycle.

The reasons for real wage rigidities are not well understood, however, and they might not properly be represented in modern business cycle models. Recently, Chari et al. (2009), for instance, have argued that standard New Keynesian (NK) models grossly fail in explaining the data along this dimension, as cyclical variations in the profit share are mostly explained by unobserved 'wage mark-up' shocks, the average size of which is very difficult to justify from micro-economic considerations.

Several studies have introduced real wage rigidities in NK models.However, these studies have invariably modeled real wage rigidities in a somewhat ad-hoc way by imposing either lagged wage 'norms' or rent-sharing over the business cycle. They report that real wage rigidities have important implications for optimal monetary policy (Blanchard and Gali, 2006; Faia, 2007) and that they improve the empirical fit of business cycle models (Christoffel and Linzert, 2006; de la Croix et al., 2006; Küster, 2007).

The present paper aims at contributing to improving the micro-foundations for real wage rigidities in NK models. It is based on the shirking models by Shapiro and Stiglitz (1987) and St Paul (1995). The paper develops a shirking model, where involuntary layoffs are a function of the change in the firm's employment. This model is embedded within a standard New Keynesian framework to study the effects of productivity and monetary policy shocks.

The shirking model assumes that the firm can control employees' effort only imperfectly. The firm must therefore create incentives for workers to avoid shirking. This is achieved by raising the real wage above the expected alternative income faced by laid-off workers, while threatening to fire shirking workers. As a result, there arises involuntary unemployment. The conjecture that such mechanism might also limit the responsiveness of real wages to cyclical variations in labour productivity has been investigated by a few studies, which have incorporated shirking into real business cycle models (Danthine and Donaldson, 1990, 1995; Gomme, 1999; Alexopolous, 2004). These studies found very small effects. They all assumed a constant separation rate and ruled out involuntary layoffs.

Yet, although firms are to some extent able to downsize their workforce from voluntary quits, involuntary layoffs do occur during downturns. Following St. Paul (1995), I model the rate of involuntary layoffs as a function of the change in the firm's employment. I will show that this feature introduces a wedge between the marginal productivity of labour and the real wage and creates a counter-cyclical wage mark-up. This arises, as workers' incentives depend on expected future wages, which are discounted with the probability of survival within the firm. The latter, however, varies with employment over the business cycle. In case of an economic upturn, for instance, the higher probability of survival raises the expected future value of employment. This leads the worker to accept a lower current wage and allows the firm to raise its profit share.

Extended by these features, the shirking model not only is capable of generating a high degree of real wage rigidity and a pro-cyclical profit share. It also replicates the lagged hump-shaped response of employment to the GDP cycle, while output per worker tends to lead the latter.

Recent empirical studies on firm-worker relationships support the idea that workers' effort is hampered by a disruption of working conditions in the firm, which tend to deteriorate during economic downturns. According to a survey by Bewley (1999), for instance, US managers explain their sluggish adjustment of wages in recessions predominantly by their concerns about a worsening in workers' morale. Franz and Pfeiffer (2006) and Agell and Bennmarker (2007) report similar findings.

The plan of the paper is as follows. Section 2 reviews stylized labour market facts for the euro area and discusses recent empirical literature on the behavioral economics of real wage rigidity Section 3 presents a Neo-Keynesian model with shirking. Section 4 reports simulation results from a calibration exercise. Section 5 concludes the paper.

2 The cyclical behavior of labour markets

In order to motivate the modelling of real wage rigidity, I briefly review stylized facts about the cyclical behavior of aggregate labour market data for the euro area. Table 1 shows sample cross correlations among the cyclical components of key series, as obtained from the approximate bandpass filter (Baxter and King, 1999).[1]

The cyclical behavior of employment and productivity matches the well-known patterns. The employment cycle lags the output one by about 1-2 quarters with a relative standard deviation of about 0.60. The labour productivity cycle, in turn, leads the output one with a slightly larger relative standard deviation of 0.68. Maximum cross correlations are high. Correspondingly, studies for U.S. data based on time series methods find that output, employment and labour productivity follow a common cycle (Krolzig et al., 2003; Rünstler, 2004).

By contrast, the cyclical component of the euro area real wage lacks a systematic association with GDP and productivity cycles. While its relative standard deviation of 0.60 is sizeable, the maximum cross correlation with the GDP cycle amounts to a mere 0.35, and the correlations with the labour productivity cycle are even lower. As a result, the labour share is counter-cyclical to output and productivity.

Studies for the U.S. find similar patterns. Using the bandpass filter, Stock and Watson (1999) report a moderately pro-cyclical wage. Studies based on other methods often find the real wage to be a-cyclical or even slightly counter-cyclical. Structural VARs, for instance, tend to deliver a negative response to monetary policy shocks (Bernanke et al., 2005). Similarly, studies based on frequency domain regressions or structural time series tend to find a-cyclical real wages (Cushing, 1990; Mocan and Topyan, 1993; Rünstler, 2004). Mocan and Topyan (1993) argue that wages are pro-cyclical in response to the supply shocks of the 1970s, but counter-cyclical after demand shocks. In a review of the literature, Abraham and Haltiwanger (1995) conclude that the findings are fragile and depend on the precise data definitions, de-trending methods used, and the time periods examined.

[1] Following Stock and Watson (1999), the bandpass filter is set to extract frequencies of 6 to 32 quarters. The data are from the ECB area-wide model database (Fagan et al., 2005).

Table 1: Sample cross moments with GDP cycle
Euro area (1970 Q1 - 2005 Q4)

	σ_r	-8	-5	-3	-2	-1	0	1	2	3	5	8
					Cross correlations with GDP							
GDP		-.32	-.12	.43	.72	.92	1.00	.92	.72	.43	-.12	-.32
Employment	.60	-.30	-.16	.20	.42	.62	.76	.82	.79	.68	.33	-.06
Labour productivity	.68	-.24	-.03	.46	.68	.82	.81	.64	.36	.04	-.48	-.44
Labour share	.80	.18	-.03	-.36	-.49	-.51	-.43	-.28	-.02	.20	.60	.56
Real wage	.60	-.02	-.10	.02	.13	.24	.33	.35	.33	.30	.25	.25
Labour force	.31	-.41	-.30	.07	.27	.44	.54	.58	.54	.45	.23	.11
				Cross correlations with labour productivity								
Real wage		.05	-.05	.04	.12	.19	.23	.19	.11	.03	.22	.41
Labour share		.37	.22	-.22	-.45	-.60	-.65	-.60	-.45	-.23	.22	.41

Column σ_r shows the standard deviation of the cyclical component relative to the GDP cycle. The remaining columns show the cross correlations $corr(y(t), x(t-s))$ of cyclical components $x(t)$ with the GDP cycle $y(t)$, (and, for the last two rows with the labour productivity cycle).

Overall, this suggests that the real wage shows little systematic association with the cycle in real activity and possibly displays different responses to supply and demand shocks.

One strand of literature emphasizes firm-worker relationships as a reason for the a-cyclicality of real wages. It is held that the firm faces difficulties with controlling worker effort, and therefore aims at raising worker motivation. This is achieved from firm-worker relationships that are perceived as fair and trustful by workers. The fact that working conditions deteriorate during economic downturns suggests that these considerations may help in explaining real wage rigidity over the business cycle.[2]

Recently, evidence for this assertion has been provided by surveys among business managers (e.g. Bewley, 1999; Franz and Pfeiffer, 2006; Agell and Bennmarker, 2007). These studies report that managers refrain from reducing the real wage in economic downturns predominantly because of concerns about a worsening in workers' effort. Managers report that cutting real wages would reduce identification with the firm, lead to disruptions in trust and cooperative behavior, and reduce worker morale. Instead of cutting wages, managers therefore prefer to adjust employment and to re-assure remaining workers about their survival in the firm, while keeping their wage at unchanged levels.

[2]These arguments have been put forward by proponents of efficiency wage (Akerlof, 1982) and implicit contract theories (see, e.g., Azariadis and Stiglitz, 1983). Experimental evidence on the effects of worker motivation on productivity actually dates back to the Hawthorne experiments (Mayo, 1933). For more experimental evidence see, e.g., Fehr et al. (1993).

Bewley (2007, p. 162f) summarizes further evidence from 12 respective studies. Managers report, for instance, that worker supervision is kept at low levels because it is regarded as expensive. Maintaining 'fair' internal pay structures is seen as important for worker motivation. This implies that wage increases are negotiated for the workforce as a whole such that internal pay structures are largely maintained. This also prohibits potential underbidding by unemployed workers.

Another important element is the significant wage losses that workers face in jobs subsequent to involuntary displacement in the previous job. A number of studies finds wage losses of displaced workers to amount to 20% or more on average and to persist over a period of about 3-5 years. For the U.S. and Canada wage losses are found to prevail for all skill levels across different industries (e.g. Jacobson et al., 2002; Kletzer, 2004; Hu, 2006; Morisette et al., 2007). For Germany, Burda and Mertens (2001) report similar losses, but with the exception of low-skill workers.

Wage losses may be related to the lack of firm-specific skills and to seniority pay structures, which imply lower wages for new firm entrants. Note that there also exist other sources of costs, when starting a new job, such as additional effort needed to acquire job-specific skills, and possibly relocation or a need for commuting, or less job satisfaction.

3 A shirking model with endogenous layoffs

I consider a basic New Keynesian economy with indivisible labour and a fixed capital stock. The economy is inhabited by two types of households, i.e. a measure j of entrepreneur households in $(0, 1)$ and a measure ζ of workers in $(0, 1)$. Entrepreneur households own the fixed capital stock $\overline{k}$ and obtain rents from the latter, but do not provide labour services. They can save by holding government bonds. Workers, in turn, obtain their income by providing labour services. They may be either employed or unemployed. Workers do not save.

Workers are hired by intermediate goods firms, which operate on a fully competitive markets. When choosing employment and the real wage, firms face an efficiency wage constraint, which they have to satisfy in order to avoid shirking by workers. Price rigidities emerge from final goods producers, which assemble differentiated consumption goods using a linear technology and face Calvo price setting constraints.

The distinction between entrepreneur households and workers, together with the restriction that workers are not allowed to save is due to Gomme (1999). It is required to avoid heterogeneity in individual wealth positions that arise from different unemployment histories of individual workers. Such heterogeneity almost necessarily arises in NK models with unemployment. Alternatively, studies have assumed perfect insurance against unemployment in consumption, while ignoring such insurance in worker's incentives when in comes to wage setting (Christoffel and Linzert, 2006; Küster, 2007).[3]

The government performs two functions. First, it runs an unemployment insurance scheme, which is self-financing in the steady state. For this purpose, the government taxes the gross wage w_{it} at rate τ and pays unemployment benefits b to unemployed. This leaves the worker employed in firm i with net real wage $a_{it} = w_{it}(1-\tau)$. Second, the government issues bonds B_t. A monetary authority issues money M_t.and sets the nominal interest rate r_t.

3.1 Entrepreneur households

Entrepreneur household $j \in (0,1)$ maximizes lifetime utility

$$V_{jt} = \max \left[g_{C,t} \left(\ln\left(c_{jt} - \kappa c_{t-1}\right) + \Lambda \log \frac{M_{jt}}{P_t} \right) + \beta \mathbb{E}_0 V_{jt+1} \right]$$
$$g_{C,t+1} = g_{C,t}^{\rho_c} \varepsilon_t^c, \qquad \ln \varepsilon_{C,t} \sim \mathbb{N}(0, \sigma_C^2)$$

choosing a consumption bundle c_{jt}, nominal money holdings M_{jt} and bonds B_{jt} . Parameter $0 \leq \kappa < 1$ describes the degree of external habit formation in consumption, i.e. the dependence of utility on past aggregate consumption c_t. Parameter $\Lambda > 0$ denotes the the weight of money holdings in utility. $g_{C,t}$ is a serially correlated shock to household preferences.

Household j is subject to the budget constraint

$$c_{jt} + \frac{M_{jt}}{P_t} + \frac{B_{jt}}{P_t} = \frac{\Pi_{jt}}{P_t} + \frac{M_{jt-1}}{P_t} + (1 + r_t)\frac{B_{jt}}{P_t}$$

where Π_{jt} are nominal firm profits and P_t and r_t denote the aggregate price level and nominal interest rate on bonds, respectively.

[3] The alternative approach gives rise to very similar outcomes. In terms of models it would merely change the aggregate resource constraint (section 4) to $y_t = c_t$.

The consumption bundle is a CES aggregate of differentiated final goods, indexed by k,

$$c_{jt} = \left(\int_0^1 c_{jkt}^{1-1/\nu} \right)^{\frac{\nu}{\nu-1}} \qquad P_t = \left(\int_0^1 P_{kt}^{1-\nu} \right)^{\frac{1}{1-\nu}}, \quad \nu > 1$$

The first order conditions are

$$\begin{aligned}
g_{C,t} \left(c_{jt} - \kappa c_{t-1}\right)^{-1} &= \beta \mathbb{E}_t \left[g_{C,t+1} (1 + r_{t+1}) \frac{P_{t+1}}{P_t} \left(c_{jt+1} - \kappa c_t\right)^{-1} \right] \\
\frac{M_{jt}}{P_t} &= \Lambda \left[c_{jt} - \kappa c_{t-1}\right]^{\xi} \mathbb{E}_t \frac{1 + r_{t+1}}{r_{t+1}} \\
c_{jkt} &= \left(\frac{P_{kt}}{P_t} \right)^{-\nu} c_{jt}, \quad k \in [0, 1]
\end{aligned}$$

3.2 Workers

The economy is inhabited by a measure ζ of workers in $(0, 1)$ with utility

$$u(c_{\zeta t}, e_{\zeta t}) = \chi \ln c_{\zeta t} + (1 - \chi) \ln(\mu - e_{\zeta t}) \qquad 0 < \chi < 1$$

where $c_{\zeta t}$ and $e_{\zeta t}$ denote consumption and effort, respectively. This is the case of a CRRA utility with relative risk aversion $\xi = 1$. I limit the exposition in the paper to this case. Annex A discusses the general case of $\xi > 1$.

In each period t, a worker ζ is either employed in an intermediate goods firm i or unemployed. Employed workers receive net real wage a_{it}, while unemployed workers receive fixed real unemployment benefits b_t from the government. Workers do not save. However, employed workers decide upon the effort they display in their job.

At the end of period t a worker leaves the workforce with probability $1 - x$. At the same time, a new cohort of $1 - x$ unemployed workers enters the workforce.

The incentive compatibility constraint. At the beginning of period t firm i offers the same labour contract to all workers. The contract extends over period t and foresees a certain net real wage a_{it} and a fixed level of effort $\overline{e} = 1$.

Thereafter, production starts and the individual worker ζ employed in firm i decides upon his effort $e_{\zeta t}$. A worker may either choose to work, i.e. to display effort $e_{\zeta t} = 1$ or to shirk, $e_{\zeta t} = 0$. The firm assesses the worker's effort from a fixed technology such that the individual worker faces probability γ of being detected when shirking. In such case, he is fired at the end of period t. Otherwise, the worker faces probability ϕ_{it+1} to remain employed in firm i in period $t+1$.

Denote with V^N_{it+1} the value of being employed in firm i period $t+1$ and with V^A_{t+1} the value of being laid off at the end of period t, respectively. The employed worker will choose effort $e_{\zeta t} = 1$ if and only if the value of doing so,

$$V^N_{it} = u(a_{it}, 1) + \beta x \mathbb{E}_t \left[\phi_{it+1} V^N_{it+1} + (1 - \phi_{it+1}) V^A_{t+1}\right] \tag{1}$$

exceeds the value of shirking,

$$V^S_{it} = u(a_{it}, 0) + \beta x(1-\gamma) \mathbb{E}_t \left[\phi_{it+1} V^N_{it+1} + (1 - \phi_{it+1}) V^A_{it+1}\right] + \beta x \gamma \mathbb{E}_t V^A_{t+1} \tag{2}$$

The condition $V^N_{it} \geq V^S_{it}$ gives the incentive compatibility constraint (ICC)

$$\beta x \gamma \mathbb{E}_t \phi_{it+1} [V^N_{t+1} - V^A_{t+1}] \geq \omega \tag{3}$$

where $\omega = -(1-\chi)\ln((\mu - 1)/\mu) > 0$.

The above ICC is a version of the shirking model due to Shapiro and Stiglitz (1987). It relates the future expected excess value of employment, $V^N_{it+1} - V^A_{t+1} > 0$ *negatively* to the probability $x\phi_{it+1}$ of survival in the firm. The intuition is that the worker avoids shirking if the expected value of rents from future employment increases, the latter being discounted with the probability of survival in the firm.

3.3 Worker flows and survival probabilities

The labour market operates as follows. At the end of period t, intermediate goods firm i, first, decides on the desired level of employment n_{it+1} for period $t+1$. Second, a random re-allocation of workers across firms takes place, which reflects voluntary moves to other jobs. At the same, a fraction $1 - x$ of workers leave the labour force.

As a net outcome of the workers' flows, the workforce of firm i evolves with $Z^{-1}_{it} n_{it}$, where Z^{-1}_{it} is a random variable over support $Z^{-1}_{it} > 0$ with $\mathbb{E} Z^{-1}_{it} = x$. Note that this includes the possibility $Z^{-1}_{it} > 1$, i.e. that the workforce of firm i increases due to reallocation.

Thereafter, the firm may either displace redundant or hire additional workers to achieve the desired level of employment for period $t+1$. Profit maximization implies that the firm will lower the wage w_{it+1} until the equality sign holds in equation (3). Moreover, the firm can reduce the wage by increasing ϕ_{it+1}. Hence, it will commit to never replace a non-shirking worker (i.e. to fire a worker and to hire another one in his place). The individual worker therefore faces a probability of remaining employed of $\phi_{it+1} = 1$ if $n_{it+1} > Z_{it}^{-1} n_{it}$ and $\phi_{it+1} = n_{it+1}/(Z_{it}^{-1} n_{it})$ otherwise.

Hence, the expected probability of lay-off is given by

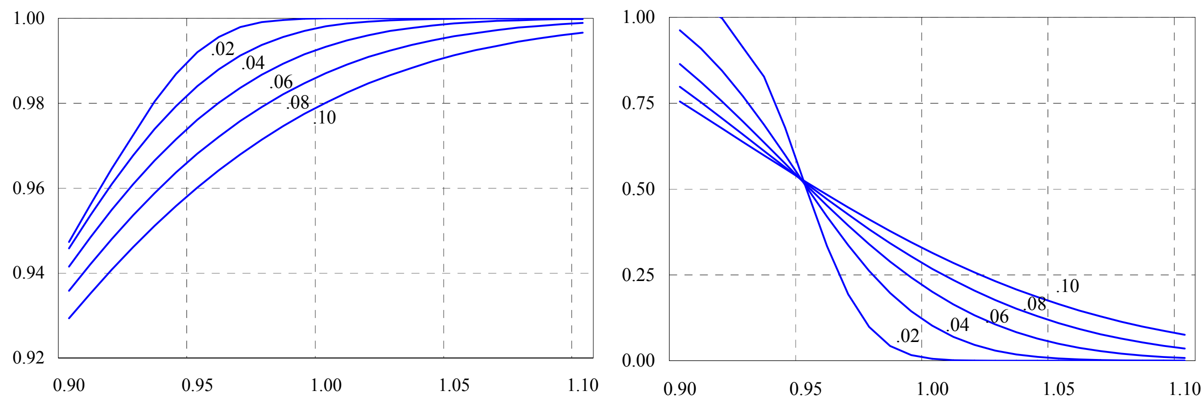
$$\mathbb{E}_t \phi_{it+1} = \phi(\frac{n_{it+1}}{n_{it}}) \equiv \mathbb{E}_Z \min\left[Z_{it}\frac{n_{it+1}}{n_{it}}, 1\right]$$

As shown in Annex C, the expectation can be calculated as

$$\phi(s) = 1 - \int_0^{s^{-1}} (1 - zs)\, dF(z) \tag{4}$$

where $F(z)$ is the distribution function of Z. It holds $\phi(0) = 0$, $\phi(\infty) = 1$, $\phi'(s) > 0$ and $\phi''(s) < 0$.

Figure 1: $\phi(s)$ and $\Psi(s)$



Based on a lognormal distribution for Z_{it}, Figure 1 shows the shapes of functions $\phi(s)$ and $\Psi(s) = \phi'(s)s/\phi^2(s)$ in the neighbourhood of $s = 1$. The graphs are based on a value of $x = 0.95$ and use various values of standard deviation σ of the distribution, ranging $\sigma = 0.02$ from to $\sigma = 0.10$. As $\sigma \to 0$, function $\phi(s)$ converges towards a kinked function with $\phi(s) = s$ for $s < x$ and 1 otherwise. Note that the slope of $\Psi(s)$ at $s = 1$ remains largely unaffected by the value of σ.

Overall, the above random allocation of workers allows for obtaining the probability of involuntary layoffs as a smooth function of the change in the firm's employment. Alternatively, one might motivate function $\phi(s)$ from idiosyncratic shocks to the productivity of individual workers or of job varieties (St. Paul, 1995).

3.4 Intermediate goods firms

Firm i produces an intermediate good y_{it} from production function

$$
\begin{aligned}
y_{it} &= g_t(h_{it}\overline{k}_i)^{\alpha}(e_{it}n_{it})^{1-\alpha} - \Gamma(h_{it}) && (5)\\
g_{t+1} &= g_t^{\rho}\varepsilon_{g,t}, \qquad \ln\varepsilon_{g,t} \sim \mathbb{N}(0,\sigma_g^2) && (6)\\
\Gamma(h) &= \frac{\delta_0}{\delta+1}(h^{\delta+1}-1)
\end{aligned}
$$

with variable capital utilization h_{it}. The variable g_t represents the state of technology, which is common across firms and assumed to vary exogenously over time according to equation (6). The firm pays gross real wage w_{it} to a worker.

By inserting equation (1) shifted one period forward into ICC (3) one can express the latter as

$$\omega + \beta x \mathbb{E}_t \phi(\frac{n_{it+1}}{n_{it}})\,[\chi\gamma \ln w_{it+1} + \chi\gamma\ln(1-\tau) + \omega + \gamma(1-\chi)\ln\mu - \gamma f_{t+1}] \geq 0 \qquad (7)$$

$$f_{t+1} = V_{t+1}^A - \beta x \mathbb{E}_{t+1} V_{t+2}^A \qquad (8)$$

The firm maximizes its value

$$W(n_{it}) = \max\left[q_t g_t(h_{it}\overline{k}_i)^{\alpha} n_{it}^{1-\alpha} - n_{it}w_{it} + \beta\mathbb{E}_t W(n_{it+1})\right] \qquad (9)$$

with respect to state n_{it} and control variables w_{it+1} and h_{it} subject to the ICC (7).

q_t denotes the price of the intermediate good in terms of the final good. The firm takes as given technology g_t, the relative price of intermediate goods q_t , the tax rate τ on wages, and the current and expected future values of the alternative option V_t^A in ICC (3). In the following I assume that the constraint $n_{it} \leq 1$ is not binding in the firm's hiring plan.

The first-order conditions are given by (see Annex A)

$$\alpha q_t \frac{y_{it}}{h_{it}} = \delta_0 h_{it}^{\delta} \tag{10}$$

$$(1-\alpha) q_{t+1} \frac{y_{it+1}}{w_{it+1} n_{it+1}} = 1 + \frac{\omega}{\chi\gamma} \mathbb{E}_{t+1} \left(\Psi(\frac{n_{it+2}}{n_{it+1}}) \frac{n_{it+2} w_{it+2}}{n_{it+1} w_{it+1}} - \beta^{-1} \Psi(\frac{n_{it+1}}{n_{it}}) \right) \tag{11}$$

where $\Psi(s) = \phi'(s)/\phi^2(s)$.

From equation (11), the wage mark-up varies over the cycle as a function of expected future employment. Equation (11) bears some resemblance to a labour demand equation that is obtained from quadratic labour adjustment costs (Christiano et al., 1995). In this sense, the ICC (7) may be viewed as creating labour adjustment costs in terms of a requirement to set the current real wage at a certain level.

Evidently, a constant separation rate, $\phi(s) = \phi_0$ implies $\Psi(s) \equiv 0$, and the wage mark-up is therefore identical to zero. This observation plainly explains the failure of earlier shirking models to generate real wage persistence (Danthine and Donaldson 1990, 1995; Gomme, 1999).

Both n_{it} and w_{it} are predetermined in the firm solution. This is a consequence of the fact that worker utility is additive in consumption and leisure. Annex C derives the solution for CRRA utility with $\xi > 1$. The real wage then enters the ICC both in periods t and $t+1$ and becomes non-predetermined, while the solution to the firm problem is state-dependent. This adds some complexity, but little is gained in the model properties. Hence, I stick to the simpler case of $\xi = 1$.

3.5 Final goods producers and price setting

Final goods producers purchase a certain amount $y_{k.t}$ of intermediate goods at price $q_t P_t$ and produce differentiated goods y_{kt}^F from a linear technology, $y_{kt}^F = y_{kt}$. Final goods producers face the standard Calvo price setting constraint. That is, final goods firm k faces a probability θ of being able to reset its price in period t. In case it gets the chance it sets p_t^* to maximize

$$\sum_h \theta^h \mathbb{E}_t \left[Q_{t,t+h} \left(p_t^* - q_{t+h} P_{t+h} \right) y_{k.t+h|t} \right]$$

subject to the sequence of demand constraints

$$y_{t+h|t} = \left[p_t^* / P_{t+h} \right]^{-\nu} y_{t+h}$$

where $Q_{t,t+h} = \beta^h \left(C_t^{-1}P_t\right)/(C_{t+h}^{-1}P_{t+h})$ denotes the stochastic discount factor. The firm problem gives rise to the well-known first order condition

$$\sum_h \theta^h \mathbb{E}_t \left[Q_{t,t+h} y_{k.t+h|t} \left(p_t^* - \frac{\vartheta}{\vartheta - 1} q_{t+h} P_{t+h}\right)\right] = 0 \tag{12}$$

Further, the aggregate price level evolves according to

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\nu} = \theta + (1-\theta)\left(\frac{p_t^*}{P_{t-1}}\right)^{1-\nu}$$

3.6 Equilibrium

I consider a symmetric equilibrium with identical intermediate and final goods firms, i.e. $n_{it} = n_t$, $w_{it} = w_t$, $h_{it} = h_t$, $y_{kt}^F = y_t$ for all t. Similarly, I assume identical ent repreneur households, $c_{jt} = c_t$. In the following I denote with $\overline{x}$ the steady state level of variable x_t.

The model is closed from the aggregate resource constraint, a monetary policy rule, the self-financing condition for the unemployment insurance scheme, and a labour market equilibrium condition, which determines the excess value of employment against the value of the alternative option, $V_{t+1}^N - V_{t+1}^A$.

Resource constraint. I assume that the unemployment insurance run by the government is financed from taxing wages w_{it} with fixed rate τ, such that it is self-financing in the steady state, with steady state unemployment rate $1 - \overline{n}$. .This implies the aggregate resource constraint

$$\begin{aligned} y_t &= c_t + (1-\tau) w_t n_t + b(\overline{1} - n_t) \\ \tau &= (1-\overline{n}) b / (\overline{nw}) \end{aligned}$$

Monetary policy rule. The monetary authority sets interest rates according to the Taylor rule

$$r_t = \beta^{-(1-\rho_r)} r_{t-1}^{\rho_r} \pi_t^{\rho_\pi} \left(\frac{y_t}{\overline{y}}\right)^{\rho_y} \varepsilon_t^m, \qquad \ln \varepsilon_t^m \sim \mathbb{N}(0, \sigma_m^2) \tag{13}$$

where $\pi_t = P_t/P_{t-1}$ is gross inflation. The rule implies a steady level of inflation of $\overline{\pi} = 1$.

Labour market equilibrium. Finally, a labour market equilibrium condition determines the excess value of employment against the value of the alternative option, $V_{t+1}^N - V_{t+1}^A$. The value of the alternative option in period $t+1$ is defined as

$$V_{t+1}^{A} = s_{t+1}(V_{t+1}^{N} - \eta) + (1 - s_{t+1})V_{t+1}^{U} \tag{14}$$

$$V_{t+1}^{U} = \chi \ln b + \beta x \mathbb{E}_{t+1} V_{t+2}^{A} \tag{15}$$

where s_{t+1} denotes the probability of finding a job, faced by a worker that is unemployed at the beginning of period $t+1$. V_{t+1}^{U} is the value of being unemployed in period $t+1$. Constant $\eta > 0$ denotes the fixed costs associated with starting a new job out of involuntary unemployment.

Using equations (1) and (3) the excess value $V_{t+1}^{N} - V_{t+1}^{A}$ can be expressed as (see Annex D)

$$V_{t+1}^{N} - V_{t+1}^{A} = \eta s_{t+1} + (1 - s_{t+1}) \left(\chi \ln \frac{a_{t+1}}{b} + \frac{1-\gamma}{\gamma} \omega \right) \tag{16}$$

Equation (16) shows that, for a given wage and with $\eta = 0$, the excess value of employment over the alternative option, $V_{t+1}^{N} - V_{t+1}^{A}$, moves counter-cyclically to employment, as re-employment probability s_{t+1} is pro-cyclical. Start-up costs $\eta > 0$ therefore act to reduce the cyclical volatility of the excess value.

It remains to specify s_{t+1}. I assume that vacancies and job searchers are perfectly matched and that firms choose randomly among job applicants. The number of searchers is then found from the sum of laid-off workers, $(1 - \phi(n_{t+1}/n_t))n_t$, the previously unemployed $x(1 - n_t)$ and the new entrants into the labour force, $1 - x$. The number of hires is given by $n_{t+1} - (\phi(n_{t+1}/n_t) + 1 - x)\, n_t$. Hence,

$$s_{t+1} = \frac{n_{t+1} - (\phi(n_{t+1}/n_t) - 1 + x)\, n_t}{1 - (\phi(n_{t+1}/n_t) - 1 + x)\, n_t}$$

Steady state. I conclude this section with two observations on the steady state. First, the condition $\eta < \overline{V}^{N} - \overline{V}^{A}$ is required to ensure that the excess value is negatively related to employment in the steady state $\partial(\overline{V}^{N} - \overline{V}^{A})/\partial\overline{n} < 0$. It is then easily established that the steady state labour supply schedule is upward sloping, $\partial\overline{w}/\partial\overline{n} > 0$.[4]

Second, the steady state mark-up of wages over marginal costs depends on the ICC. The steady state labour share is found with

$$\frac{\overline{wn}}{\overline{y}} = (1-\alpha)\nu \left[1 - \frac{\omega}{\chi\gamma} \left(\beta^{-1} - 1 \right) \Psi(1) \right]^{-1}$$

[4] The case of downward sloping labour supply, $\eta > \overline{V}^{N} - \overline{V}^{A}$, gives rise to indeterminate equilibria (compare Farmer, 1999).

The wage mark-up, hence, depends on worker utility and on the features of the firm-employee relationship, as embodied in parameters related to ICC (3). The labour share declines with higher γ and a lower disutility of work, as embodied in parameters χ and μ.

4 Simulation results

This section presents simulation results from a model calibration exercise. More precisely, it discusses impulse responses and stochastic simulations for shocks to consumption preferences of 1% and total factor productivity of 0.25%. The results for the monetary policy shock are almost identical to the preference shock and are therefore not shown.

Parameter values. Parameter values are shown in Table 1. I set the steady state value of employment $\overline{n}$ at 0.93, implying an unemployment rate of 7%, and derive parameter $\mu > 1$ such that the steady state conditions are satisfied. Unemployment benefits b are set to achieve a steady–state replacement ratio of $b/\overline{a} = 0.45$ (Nickell and Nunziata, 2005).

Probability γ is set to a value of 0.75. As regards labour market transition probabilities, recent studies use a quarterly separation rate of about 10% (e.g. Lubik and Krause, 2007). Fallick & Fleischmann (2004) report that about 40% of separating workers move directly to another firm, while another 40% leave the labour force and 20% move into unemployment. I assume that two third of the moves into unemployment and out of the labour force are voluntary. These assumptions result in a value of $x = 0.95$ and a steady-state probability of involuntary layoffs of 2% per quarter, which is achieved from $\sigma = 0.10$ in equation (4).[5] The benchmark calibration of parameter η is based on the rather cautious assumption of a wage loss of about 10%, which persists for 2 years. This results in $\eta = 0.6$.[6]

Parameters related to entrepreneur utility and firms are set to standard values used in the literature. As to function $\Gamma(h)$, I assume a steady state value of $\overline{h} = 1$ and set δ_0 accordingly. For the Calvo parameter I follow Gali (2008, p.52), while the parameters of the monetary policy reaction function

[5] Fallick and Fleischmann (2004) report that worker re-allocation drops markedly in recessions, which implies a shift to the right of functions $\phi(s)$ and $\Psi(s)$. This would amount to a sharper effect of the efficiency wage effect in recessions, i.e. downward real wage rigidity.

[6] The value of η is calibrated from the difference in the steady state present values of the full and reduced wages, $\gamma = \sum (\beta x \phi(1))^i (u(a,e) - u(\theta_i a, e))$, where $\theta_i = 0, 1$ denotes the percentage wage loss in period i.

are taken from Christoffel et al. (2008).

Table 2: Parameter values

Entrepreneur utility			Worker utility			Labour market				
β	κ	ν	χ	ξ	μ	σ	x	γ	b	η
0.990	0.400	6.000	0.600	1.00	1.532	0.100	0.950	0.750	0.238	0.600
Firms			**Monetary Policy**				**Shock persistence**			
α	δ	θ	ρ_r	ρ_π	ρ_y		ρ_g	ρ_c	ρ_b	
0.360	0.200	0.750	0.800	0.300	0.050		0.800	0.800	0.800	

Benchmark model. To provide a benchmark I will also present results for a NK model, which is identical to the shirking model with the exception that it features competitive labour markets and a standard labour supply equation derived from household utility. The model equations are described in annex F.

Preference shocks. Figure 2a shows the impulse responses to a consumption preference shocks $\varepsilon_{C,t} = 0.01$. In both models, employment displays a hump-shaped response. Variable capacity utilization allows for a positive response of output in period 1. Hence, labour productivity jumps upwards along with output and the response of employment lags the one of output.

As to the real wage, the benchmark NK model is sharply at odds with the empirical facts. It delivers a pronounced *increase* in the labour share of income, as the real wage shifts up due to nominal rigidities in price setting. In the shirking model, the real wage moves sluggishly and initially shows even a small negative response. As a result, the labour share largely mirrors the shifts in labour productivity. Initially, it jumps downwards and gradually returns to baseline.

Table 3 shows the cross correlations of the labour market series with output from stochastic simulations. For the shirking model, they match their empirical counterparts shown in Table 1 pretty closely. This holds in particular for the phase shifts relative to output. Employment tends to lag output, while productivity and the labour share show a lead. Similarly, the lagged response of the real wage to output shown in Table 1 is replicated by the simulations. One dimension along which the model does not fully replicate the stylized facts in Table 1 is the volatilities of the real wage and

Fig 2a: Impulse responses for preference shock

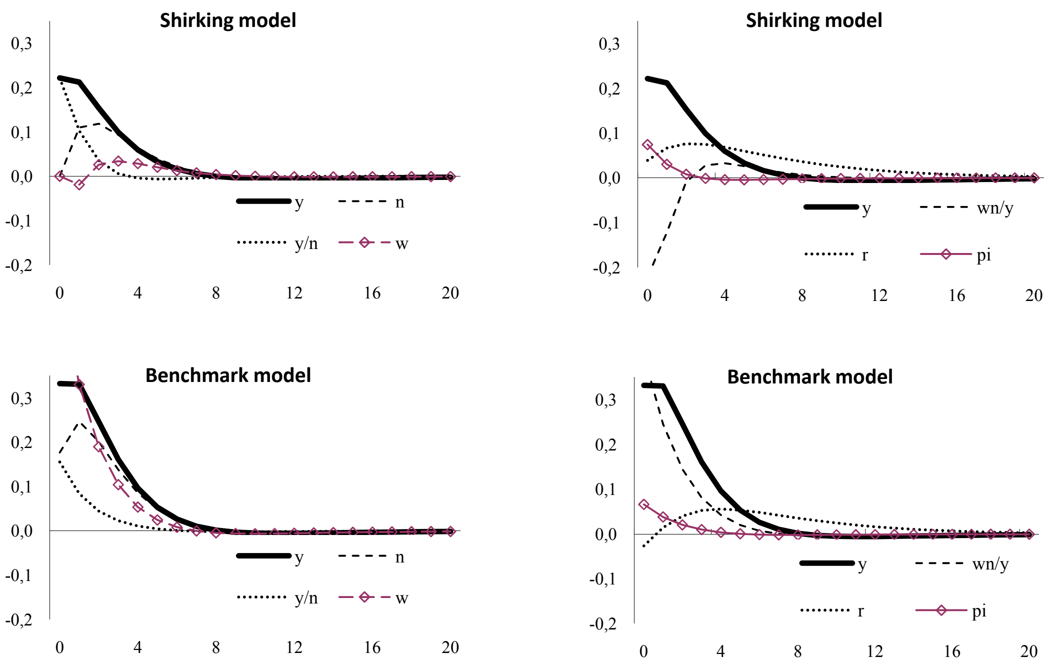
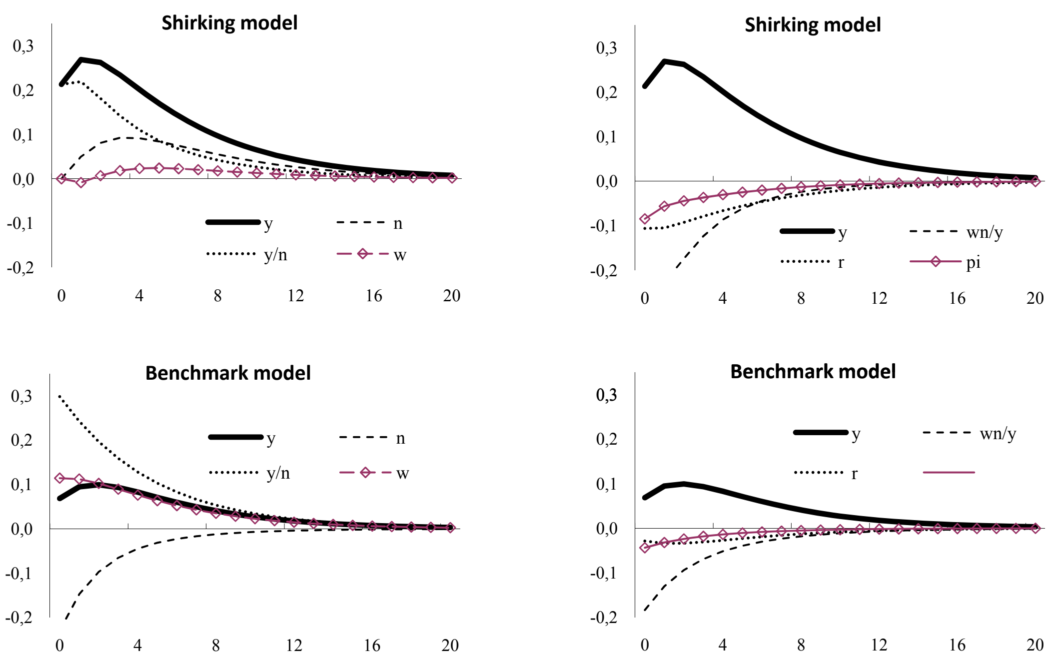


Fig 2b: Impulse responses for TFP shock



the labour share, which are smaller in the simulations, while the cross correlations with output are higher.

The behavior of inflation and interest rates is similar in both models. Lubik and Krause (2007) have pointed out that real wage rigidities do not necessarily have strong effects on inflation dynamics. The reason is that the labour share wedge in equation (11) does not reflect shifts in the marginal cost of labour.

Table 3: Simulated cross moments with GDP cycle (preference shock)

	std	-8	-5	-3	-2	-1	0	1	2	3	5	8
GDP	1.00	-.04	.05	.31	.52	.78	1.00	.78	.52	.31	.05	-.04
Employment	.57	-.04	-.01	.14	.29	.51	.76	.97	.83	.59	.18	-.05
Labour productivity	.68	-.02	.08	.34	.53	.73	.85	.33	.09	-.03	-.07	-.02
Labour share	.71	.01	-.08	-.33	-.50	-.67	-.75	-.20	.13	.23	.17	.02
Real wage	.16	-.04	-.04	-.02	.03	.12	.28	.50	.88	.82	.41	.01

Column std shows the standard deviation of the series relative to output. The remaining columns show the cross correlations $corr(y(t), x(t-s))$ of cyclical components $x(t)$ with the output cycle $y(t)$.

TFP shock. The responses to a shock to total factor productivity are shown in Figure 2b. As a consequence of nominal rigidities in price-setting, employment shows a weak response in both models. This effect is well-known (Gali,1999). In the benchmark model employment actually declines sharply, whereas it rises slightly in the shirking model. Consequently, output increases less in the benchmark model, and the rise in labour productivity is much larger in relative terms. The shirking model again delivers a very muted wage response. In the benchmark model, wages increase as much as output. Still, due to the nominal rigidities, the increase is as smaller as compared to labour productivity. Hence, the labour share responds counter-cyclically in both models. Because of the different patterns in labour productivity, the response in terms relative to output is actually smaller in the shirking model.

Sensitivity to parameters. Table 4 reports some sensitivity analysis, i.e. relative standard deviations to GDP for alternative values of the labour market parameters. The results are shown for the preference shock. The volatilities of the labour market series remain largely unaffected by variations in x and σ However, they depend on γ and η. For lower values of either γ or η, the volatility of employment declines, whereas the real wage becomes definitely pro-cyclical. The labour share remains counter-cyclical only over a short period. Higher values of either γ or η, in turn, lead to highly volatile employment, a muted response of labour productivity, and strongly counter-cyclical wages.

Table 4: Relative standard deviations for alternative parameter values (preference shock)

	x		σ		η		γ	
	.92	.98	.05	.15	.40	.80	.60	.90
Employment	.59	.53	.51	.60	.38	1.00	.42	1.00
Labour productivity	.65	.69	.71	.66	.76	.47	.75	.50
Labour share	.67	.74	.68	.76	.82	.99	.72	1.10
Real wage	.14	.18	.04	.26	.89	.91	.62	.92
Corr (y,w)	.38	.20	.73	.01	.77	-.87	.74	-.85

The table shows the standard deviations of series relative to output. The last row, corr(y,w), shows the contemporaneous correlation between output and the real wage.

5 Concluding remarks

Lucas (1987: 53) proclaims that ' .. we want a theory about unemployed people, not unemployed hours of labour services; about people who look for jobs, hold them, loose them, people with all the attendant feelings that go along with these events'. Arguably, a theory, which fulfils such claim, should take account of involuntary unemployment and imperfect unemployment insurance. In the context of such approach, the individual workers' discount factors in evaluating the future values of employment and unemployment, respectively, depend on labour market transition probabilities.

Based on the shirking model of Shapiro and Stiglitz (1987) and its extension due to St. Paul (1995) this paper has argued that the cyclical variation in worker displacement rates may have important effects on employed workers incentives. It is sometimes held that cyclical variation in separation rates can be ignored, as the major cyclical variation in employment is due to hiring (e.g. Lubik and Krause, 2007). The latter still holds true in the present model. However, the effects of variations in worker displacement on incentives nevertheless matter.

The solution to the firm's problem generates a counter-cyclical mark-up of the real wage over the marginal productivity of labour. The model appears capable of re-producing the a-cyclicality of the real wage that is found in the data. I have also argued that the mechanisms present in the model are supported by empirical findings from behavioral economics (Bewley, 1999, 2007). Compared to other work about modelling real wage rigidites in business cycle models, the efficiency wage approach provides more detailed micro-foundations. Other studies have imposed wage norms or rent-sharing and thereby miss some implications of efficiency wages for business cycle dynamics such as strong lags in the response of wages. Further, steady-state unemployment can be modelled in less detail.

There arise various routes for further research. First, the principle may also apply to wage bargaining models. Second, it seems of interest to combine the present model with search theories of the labour market in order to model the relationship between aggregate labour market conditions, worker flows and the excess value of employment in a more precise way.

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Annex A: The intermediate firm problem

Insertion of equation (1) into (7) gives

$$\begin{aligned}
& \beta x\gamma \mathbb{E}_t\phi_{it+1}[V_{t+1} - V^A_{t+1}] - \omega \\
= \; & \beta x\gamma \mathbb{E}_t\phi_{it+1}\left(u(a_{it},1) + \beta x \mathbb{E}_{t+1}\phi_{it+2}\left[V^N_{it+2} - V^A_{t+2}\right] - V^A_{t+1}\right) - \omega \\
= \; & \beta x\gamma \mathbb{E}_t\phi_{it+1}\left(u(a_{it},1) + \beta x\left[\frac{u(a_{t+1},0) - u(a_{t+1},1)}{\gamma} - \left(V^A_{t+1} - \beta x V^A_{t+2}\right)\right]\right) - \omega
\end{aligned}$$

and, with some re-arrangement, ICC (7). Denote

$$Z_{t+1} = (\chi\gamma \ln w_{it+1} + \chi\gamma\ln(1-\tau) + \omega + \gamma(1-\chi)\ln\mu - \gamma f_{t+1}) = \gamma[V_{t+1} - A_{t+1}]$$

The derivatives of the ICC (7) are given by

$$\begin{aligned}
\frac{\partial C_t}{\partial w_{t+1}} &= \beta x\chi\phi(\frac{n_{t+1}}{n_t})w^{-1}_{t+1} \\
\frac{\partial C_t}{\partial n_t} &= -\beta x\phi'(\frac{n_{t+1}}{n_t})\frac{n_{t+1}}{n_t^2}\mathbb{E}_t Z_{t+1} = -\omega\Phi(\frac{n_{t+1}}{n_t})\frac{1}{n_t} \\
\frac{\partial C_t}{\partial n_{t+1}} &= \beta x\phi'(\frac{n_{t+1}}{n_t})\frac{1}{n_t}\mathbb{E}_t Z_{t+1} = \omega\Phi(\frac{n_{t+1}}{n_t})\frac{1}{n_{t+1}}
\end{aligned}$$

where the derivatives wrt to employment are derived from inserting for $\mathbb{E}_t Z_{t+1}$ from ICC (3). The focs are

$$\begin{aligned}
\frac{\partial W_t}{\partial w_{t+1}} &: \; 0 = -\beta n_{t+1} + \lambda_t\beta x\gamma\chi\phi(\frac{n_{t+1}}{n_t})w^{-1}_{t+1} \\
\frac{\partial W_t}{\partial n_{t+1}} &: \; 0 = \beta\mathbb{E}_t\left[(1-\alpha)\frac{y_{t+1}}{n_{t+1}} - w_{t+1}\right] + \lambda_t\frac{\partial C_t}{\partial n_{t+1}} + \beta\mathbb{E}_t\lambda_{t+1}\frac{\partial C_{t+1}}{\partial n_{t+1}}
\end{aligned}$$

2which gives

$$\lambda_t = \frac{n_{t+1}}{x\gamma\chi\phi(\frac{n_{t+1}}{n_t})}w_{t+1}$$

further

$$\begin{aligned}
\lambda_t\frac{\partial C_t}{\partial n_t} &= -\frac{\omega}{\gamma\chi}\Psi(\frac{n_{t+1}}{n_t})\frac{n_{t+1}}{n_t}w_{t+1} \\
\lambda_t\frac{\partial C_t}{\partial n_{t+1}} &= \frac{\omega}{\gamma\chi}\Psi(\frac{n_{t+1}}{n_t})w_{t+1}
\end{aligned}$$

and, finally, the expression in the main text.

Annex B: Properties of function $\phi(s)$

Survival probability $\phi(s)$ is found from

$$\phi(s) = \mathbb{E}\min\left[Zs, 1\right] = \int_0^\infty \min\left[zs, 1\right] dF(z)$$

It follows that

$$
\begin{aligned}
\phi(s) &= s\int_0^{s^{-1}} z dF(z) + \int_{s^{-1}}^{\infty} dF(z) \\
&= s\int_0^{s^{-1}} z dF(z) + 1 - \int_0^{s^{-1}} dF(z) \\
&= 1 - \int_0^{s^{-1}} (1 - zs)\, dF(z)
\end{aligned}
$$

By noting that

$$
\begin{aligned}
\frac{d}{ds}\left(\int_0^{s^{-1}} dF(z)\right) &= \frac{ds^{-1}}{ds}\frac{d}{ds^{-1}}\left(\int_0^{s^{-1}} dF(z)\right) = -s^{-2} f(s^{-1}) \\
\frac{d}{ds}\left(\int_0^{s^{-1}} z dF(z)\right) &= \frac{ds^{-1}}{ds}\frac{d}{ds^{-1}}\left(\int_0^{s^{-1}} z dF(z)\right) = -s^{-3} f(s^{-1})
\end{aligned}
$$

the derivative $\phi'(s_t)$ is found as

$$
\phi'(s) = \int_0^{s^{-1}} z dF(z) + s\frac{\partial}{\partial s}\left(\int_0^{s_t^{-1}} z dF(z)\right) + \frac{\partial}{\partial s}\left(1 - \int_0^{s^{-1}} dF(z)\right) = \int_0^{s^{-1}} z dF(z)
$$

A lognormal distribution for Z_{it} seems an appropriate choice. The density function of the lognormal $\Lambda(\mu, \sigma^2)$ is given by

$$
f(z; \mu, \sigma^2) = (\sigma z\sqrt{2\pi})^{-1} \exp\left[-\frac{(\ln z - \mu)^2}{2\sigma^2}\right]
$$

over support $z > 0$. The distribution is obtained as the exponential of a normal distribution $N(\mu, \sigma^2)$, i.e. if $y \sim N(\mu, \sigma^2)$ then $z = \exp y \sim \Lambda(\mu, \sigma^2)$. Its moments are given by

$$
\begin{aligned}
\mathrm{E}\, z &= \exp\left[\mu + \sigma^2/2\right] \\
\operatorname{var} z &= (\mathrm{E}\, z)^2 \left(\exp(\sigma^2) - 1\right)
\end{aligned}
$$

Annex C: The case of $\xi > 1$

This considers the case of

$$
u(c_{\zeta t}, e_{\zeta t}) = \frac{\left(c_{\zeta t}^{\chi}(1 - e_{\zeta t})^{1-\chi}\right)^{1-\xi}}{1 - \xi} \qquad 0 < \chi < 1
$$

with $\xi > 1$. It gives rise to the ICC

$$
(\omega - 1)\frac{a_{it}^{\chi(1-\xi)}}{1 - \xi} + \beta x\gamma\mathbb{E}_t\left[\phi_{it+1}\left(V_{it+1}^N - V_{t+1}^A\right)\right] \geq 0
$$

where $\omega = (1-\overline{e})^{(1-\chi)(1-\xi)} > 1$. This version of the ICC constains the current real wage. It relates the latter negatively to both the probability $x\phi_{it+1}$ of survival in the firm and the future expected excess value of employment, $V_{it+1}^{N} - V_{t+1}^{A} > 0$.
The firm problem then becomes (I consider a slightly simplified problem with $h_{it} \equiv 1$ and $q_t \equiv 1$.)

$$W(n_{it}) = \max \left[g_t {n_{it}}^{1-\alpha} - n_{it} w_{it} + \beta \mathbb{E}_t W(n_{it+1}) \right]$$

subject to

$$\omega_1 \frac{a_{it}^{\chi(1-\xi)}}{1-\xi} + \beta x \mathbb{E}_t \phi(\frac{n_{it+1}}{n_{it}}) \left(\omega_2 \frac{a_{it+1}^{\chi(1-\xi)}}{1-\xi} - \gamma f_{t+1} \right) \geq 0$$

where $\omega_1 = \omega - 1 > 0$ and $\omega_2 = 1 - (1-\gamma)\omega$.
The firm's problem is slightly non-standard, as the ICC now contains employment and wages both in period t and $t+1$. That is, control variable w_{it} is no longer predetermined, but there appears the expected future value of state variable n_{it} in period $t+1$, which creates state dependency. The solution to this problem has been characterised by Marcet and Marimon (1999). It is obtained from introducing a predetermined state variable λ_{it}, which represents the shadow cost of the constraint. The solution to the firm's problem can be found by splitting the ICC (7) into two functions $g_1()$ and $g_2()$, i.e.

$$\begin{aligned} g_1(a_{it}, n_{it+1}, n_{it}) + \beta \mathbb{E}_t g_2(a_{it+1}, f_{t+1}) &\geq 0 \\ g_1(a_{it}, n_{it+1}, n_{it}) &= \omega_1 \frac{a_{it}^{\chi(1-\xi)}}{1-\xi} \frac{1}{\phi(n_{it+1}/n_{it})} \\ g_2(a_{it+1}) &= x\omega_2 \frac{a_{it+1}^{\chi(1-\xi)}}{1-\xi} - \gamma f_{t+1} \end{aligned}$$

and considering the problem

$$\begin{aligned} W(n_{it}, \mu_{it}) &= \max \left(g_t \left(\overline{e} n_{it} \right)^{1-\alpha} - n_{it} w_{it} + \widetilde{\mu}_{it} g_1(a_{it}, n_{it+1}, n_{it}) + \mu_{it} g_2(a_{it}) + \beta \mathbb{E}_t W(n_{it+1}, \mu_{it+1}) \right) \\ \widetilde{\mu}_{it} &= \mu_{it+1} \end{aligned}$$

Note that the argument of function $g_2(a_{it})$ now refers to period t. Maximisation takes place with respect to the control variables n_{it+1}, w_{it}, and co-state $\widetilde{\mu}_{it} = \mu_{it+1}$ with initial conditions n_{i0} and μ_{i0}. Variables g_t, and f_t, are exogenous to the firm. The derivative of the objective function with respect to employment n_{it+1} gives the condition

$$\begin{aligned} 0 &= \widetilde{\mu}_{it} \omega_1 \frac{a_{it}^{\chi(1-\xi)}}{1-\xi} \frac{\phi'(n_{it+1}/n_{it})}{-\phi^2(n_{it+1}/n_{it})} \frac{1}{n_{it}} + \beta \mathbb{E}_t \frac{\partial W(n_{it+1}, \mu_{it+1})}{\partial n_{it+1}} \\ \frac{\partial W(n_{it}, \mu_{it})}{\partial n_{it}} &= (1-\alpha) \frac{y_{it}}{n_{it}} - w_{it} + \widetilde{\mu}_{it} \omega_1 \frac{a_{it}^{\chi(1-\xi)}}{1-\xi} \frac{\phi'(n_{it+1}/n_{it})}{-\phi^2(n_{it+1}/n_{it})} \frac{n_{it+1}}{-n_{it}^2} \end{aligned}$$

where the second line is obtained from the envelope theorem. Some re-arrangement results in the labour demand equation

$$(1-\alpha) \frac{y_{it+1}}{n_{it+1}} - \mathbb{E}_t w_{it+1} = \frac{\omega_1}{\xi - 1} \mathbb{E}_t \left(\Psi(\frac{n_{it+2}}{n_{it+1}}) \frac{n_{it+2}}{n_{it+1}} \frac{a_{it+1}^{\chi(1-\xi)} \mu_{it+2}}{n_{it+1}} - \beta^{-1} \Psi(\frac{n_{it+1}}{n_{it}}) \frac{a_{it}^{\chi(1-\xi)} \mu_{it+1}}{n_{it}} \right)$$

The derivative of the objective function with respect to wages w_{it} gives the condition (note that $\partial a_t/\partial w_t = a_t/w_t$)

$$\frac{n_{it}w_{it}}{\chi a_{it}^{\chi(1-\xi)}} = \frac{\omega_1}{\phi(n_{it+1}/n_{it})}\widetilde{\mu}_{it} + x\omega_2\widetilde{\mu}_{it-1}$$

which defines a recursion in the predetermined co-state $\widetilde{\mu}_{it}$. Equations (11) and (**??**) in the main text are obtained by replacing $\widetilde{\mu}_{it}$ with $\lambda_{it}= a_{it}^{\chi(1-\xi)}n_{it}^{-1}\widetilde{\mu}_{it}$.
For determinacy, the condition is required $\omega < (1-\gamma)^{-1}$, which implies $\omega_2 > 0$. This amounts to the assumption that the firm has sufficient control over worker effort such that the impact of the ICC in period $t+2$ on V_{it+1}^N remains contained compared to the direct effect of utility in period $t+1$.

Annex D: Labour market equilibrium condition

Equation (16) is derived as follows. Subtracting equation (14) from V_{t+1}^N gives the expression

$$V_{t+1}^N - V_{t+1}^A = \eta s_{t+1} + (1-s_{t+1})(V_{t+1}^N - V_{t+1}^U)$$

Inserting the difference of equations (1) and (15) one obtains

$$V_{t+1}^N - V_{t+1}^A = \eta s_{t+1} + (1-s_{t+1})\left\{\{u(a_{t+1}, e_{t+1}) - u(b_{t+1}, 0)\} + \beta x \mathbb{E}_{t+1}\phi_{t+2}(V_{t+2}^N - V_{t+2}^A)\right\}$$

Inserting ICC (3) shifted one period forwards for $\beta x E_{t+1}\phi_{t+2}(V_{t+2}^N - V_{t+2}^A)$ gives equation (16). For $\xi = 1$ the labour market equilibrium condition is given by

$$V_{t+1}^N - V_{t+1}^A = \eta s_{t+1} + (1-s_{t+1})\left(\chi \ln(a_{t+1}/b_{t+1}) + \frac{1-\gamma}{\gamma}\omega\right)$$

For $\xi > 1$ the condition becomes

$$V_{t+1}^N - V_{t+1}^A = \eta s_{t+1} + (1-s_{t+1})\left(\frac{(\omega_2/\gamma)a_{t+1}^{\chi(1-\xi)} - b_{t+1}^{\chi(1-\xi)}}{1-\xi}\right)$$

Annex E: Calculating the steady state

A closed-form solution for the steady state can not be obtained. However, the steady state equations can be collapsed into a single equation for employment $\overline{n}$. An interior steady state solution $0 < \overline{n} < 1$ requires certain restrictions on the parameters to be satisfied. This arises from the fact that, with $\eta > 0$, the real wage remains finite once n approaches 1.
Given the steady state value of employment, $\overline{n}$ and, hence, $\overline{q}$, insertion of the steady state ICC (3) into the steady state excess value equation (16) gives for $\overline{Z} \equiv \overline{V}^N - \overline{V}^A$

$$\overline{Z} = \eta\overline{s} + (1-\overline{s})\left(\chi \ln(\overline{a}/\overline{b}) + \frac{1-\gamma}{\gamma}\omega_1\right)$$

from which $\overline{a}$ can be derived as $\overline{Z} = \omega_1(\beta x\gamma\phi(1))^{-1}$. The steady state can be entirely expressed as a function of $\overline{n}$. One may then solve for $\overline{n}$ from the labour demand equation (11) by numerical methods. Alternatively, the steady state level of $\overline{n}$ may be fixed and parameter $\overline{e}$ is solved for from equation (11).

Annex F: Benchmark model

As a benchmark, I use a standard NK model with competitive labour markets. Workers are abolished, while households provide labour. Households maximise

$$V_{jt} = \max \left[g_t^c \left(\frac{(c_{jt+1} - \kappa c_t)^{1-\xi}}{1-\xi} - b_t \frac{n_{jt}^{\chi+1}}{\chi+1} + \Lambda \log \frac{M_{jt}}{P_t} \right) + \beta \mathbb{E}_0 V_{jt+1} \right]$$

subject to the budget constraint

$$c_{jt} + \frac{M_{jt}}{P_t} + \frac{B_{jt}}{P_t} = w_t n_{jt} + \frac{\Pi_t}{P_t} + \frac{M_{jt-1}}{P_t} + (1+r_t)\frac{B_{jt-1}}{P_t} + d_t$$

This gives rise to the same first order conditions as in the main text plus the labour supply condition

$$\begin{gathered}
g_t^c \left(c_{jt} - \kappa c_{t-1}\right)^{-\xi} = \beta \mathbb{E}_t \left[g_{t+1}^c (1+r_{t+1}) \frac{P_{t+1}}{P_t} \left(c_{jt+1} - \kappa c_t\right)^{-\xi} \right] \\
\frac{M_{jt}}{P_t} = \Lambda g_t^c \left[c_{jt} - \kappa c_{t-1}\right]^{\xi} \mathbb{E}_t \frac{1+r_{t+1}}{r_{t+1}} \\
c_{jkt} = \left(\frac{P_{kt}}{P_t} \right)^{-\nu} c_{jt}, \quad k \in [0,1] \\
b_t n_{jt}^{\chi} \left(c_{jt} - \kappa c_{t-1}\right)^{\xi} = w_t
\end{gathered}$$

Here, b_t stands for a shift in labour disutility, which is used tzo implement the labour suppply shock. Intermediate goods firms face the same problem as in (9) but without the ICC, while the real wage is taken as given. This gives rise to the standard conditions

$$\begin{aligned}
\alpha q_t \frac{y_{it}}{h_{it}} &= \delta_0 h_{it}^{\delta} \\
(1-\alpha) q_t \frac{y_{it}}{n_{it}} &= w_{it}
\end{aligned}$$

Price setting by final goods firms and the monetary policy rule are as in equations (12) and (13), respectively. Parameters related to the disutility of labour are set at $\chi = 0.5$ and $\bar{b}= 1$. All other parameters take the values shown in Table 2.

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