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Optimal Investment in R&D with International Knowledge Spillovers*

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Abstract

We provide steps towards a welfare analysis of a two-country endogenous growth model where a relatively small follower absorbs part of the knowledge generated in the leading country. To solve a suitably defined infinite-horizon dynamic optimization problem a specialized version of the Pontryagin maximum principle had to be applied. For a quite small follower, optimization produces the same asymptotic rate of innovation as the market. However, relative knowledge stocks and levels of productivity differ in the two solutions. Thus, optimal policy intervention has no effect on long-run growth rates but affects these relative levels.

Keywords: Endogenous Growth; R&D Spillovers; Absorptive Capacities; Dynamic Optimization

JEL Classification: C61; O30; O40

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1 Introduction

An endogenous growth model linking a smaller follower country to a larger, informationally autarkic leader through "absorptive capacities" enabling it to tap the knowledge stock generated by the leading country was introduced by Hutschenreiter, Kaniovski and Kryazhimskii, 1995. We will refer to this model as the "leader-follower" model. It is built along the lines of the basic endogenous growth model with horizontal product differentiation (Grossman and Helpman, 1991, Chapter 3), where technical progress is represented by an expanding variety of intermediate products. The leader-follower model was symmetrized to allow for knowledge flows in both directions (Borisov, Hutschenreiter and Kryazhimskii, 1999).

Based on a comprehensive analysis of the dynamic behavior of the leader-follower model, a particular class of asymptotics was singled out. Any trajectory characterized by this asymptotics was shown to be a perfect-foresight equilibrium trajectory analogous to the one found by Grossman and Helpman for their basic model. For this type of trajectory, explicit expressions in terms of model parameters for the long-run values of key variables such as the rate of innovation, the rate of output and productivity growth, the ratio of the stocks of knowledge of the two countries, or the amounts (shares) of labor devoted to R&D and manufacturing, respectively, were provided (Hutschenreiter, Kaniovski and Kryazhimskii, 1995).

The evolution of the economy represented by this model is the result of decentralized maximizing behavior of economic agents. A perfect-foresight equilibrium trajectory generated by the model can therefore be referred to as "decentralized" or "market" solution. However, it is well-known that a market solution is not necessarily an optimal solution. Rather, non-optimality is a common outcome in the presence of externalities of some kind. According to Grossman and Helpman, 1991, in their basic model intertemporal spillovers result in a market allocation of resources which is not Pareto-optimal: too little labor is allocated to R&D. In contrast, Benassy, 1998, finds that both under-investment and over-investment in R&D (in terms of allocation of labor to R&D activities) is possible if returns to specialization are separated from the monopolistic mark up. In any case, deviations of the optimal from the market solution provide scope for welfare-enhancing policy intervention.

A welfare analysis of the leader-follower model proposed by Hutschenreiter, Kaniovski and Kryazhimskii, 1995, is missing so far. This paper reports first results of work designed to fill this gap (a complete statement is contained in Aseev, Hutschenreiter and Kryazhimskii, 2002). This work is based on optimal control theory.

This paper is organized as follows: In section 2 we set up the infinite-horizon dynamic optimization problem capturing the task of intertemporal utility maximization faced by a fictitious social planner in the follower country. The analytical elements leading to this optimization problem are briefly discussed. Section 3 contains some remarks concerning the mathematical approach and reports major results of the mathematical analysis of the dynamic optimization problem. Section 4 interprets results of the mathematical analysis and provides a comparative analysis of the market and the optimal outcome. The final section concludes.

2 The optimal control problem

In the model we analyze, an economy's homogeneous labor resources can be used in two different ways, either for manufacturing intermediate goods (which enter final output instantaneously) or in the production of blueprints for new intermediate goods (which permanently raises productivity in final goods production). The optimization problem faced by a fictitious social planner maximizing utility by allocating resources to R&D or manufacturing is the following:

$$\max J\left(n^B(.), L_n^B(.)\right) =$$

$$= \int_0^\infty e^{-\rho t}\left(\left(\frac{1}{\alpha}-1\right)\log n^B(t) + \log\left(L^B - L_n^B(t)\right)\right) dt, \tag{1}$$

$$\dot{n}^B(t) = \frac{L_n^B(t)}{a}\left(n^B(t) + \gamma n^A(t)\right), \tag{2}$$

$$\dot{n}^A(t) = \bar{g}^A n^A(t), \tag{3}$$

$$n^B(0) = n_0^B, \qquad n^A(0) = n_0^A, \tag{4}$$

$$L_n^B(t) = \left[0, L^B\right). \tag{5}$$

Let us state for now that the model parameters $\rho, \alpha, L^B, a, \gamma, \bar{g}^A$ are all positive. Their meaning, as well as that of the variables, will be made clear in what follows. Also note that the objective function (1) is the same as in the social planning problem formulated by Grossman and Helpman, 1991. Let us first comment on this objective function. Recall that production of final output is represented by a Dixit - Stiglitz - Ethier production function $Y^B(t)$ where final output is produced by a set of differentiated intermediate goods (Dixit and Stiglitz, 1977, Ethier, 1982)

$$Y^B(t) = \left[\int_0^{n^B(t)} x(j)^\alpha dj\right]^{1/\alpha}, \tag{6}$$

where $n^B(t)$ is the number of these goods invented up to time t and $x(j)$ represents the output of intermediate good of variety j. The parameter $0 < \alpha < 1$ is related to the constant elasticity of substitution $\epsilon = 1/(1-\alpha)$. Grossman and Helpman, 1991, also provide a dual interpretation of function $Y^B(t)$ as an index of utility (the "love of variety" approach) which we will not take up here. See on this issue Barro and Sala-i-Martin, 1995.

It is a well-known feature of the basic Grossman - Helpman model that in a momentary, symmetric equilibrium (an efficient static allocation of resources at any instant of time), all types of intermediate goods are produced in the same quantities. If $x^B(t)$ denotes this uniform output per brand, aggregate output of intermediates is given by $X^B(t) = n^B(t)\, x^B(t)$. Consequently, for the production function $Y^B(t)$, final output at time t is given by

$$Y^B(t) = \left(n^B(t)\right)^{1/\alpha} x^B(t) = \left(n^B(t)\right)^{1/\alpha - 1} X^B(t). \tag{7}$$

Thus, total factor productivity (TFP) measured at time t is an increasing function of the the number of intermediate goods invented in the country so far which, in turn, is taken to represent the country's current stock of knowledge:

$$\frac{Y^B(t)}{X^B(t)} = \left(n^B(t)\right)^{1/\alpha - 1}. \tag{8}$$

With steady growth, characterized by a constant allocation of labor to manufacturing and R&D, the growth rate of final output and TFP is identically $(1/\alpha - 1)\, \bar{g}^B(t)$, where $\bar{g}^B(t)$ denotes the steady-state rate of growth of the country's knowledge stock.

In the basic Grossman - Helpman model, each intermediate good is produced by a constant-returns-to-scale technology where one unit of labor is required to turn out one additional unit of output. Consequently, aggregate output of intermediate goods equals total labor allocated to manufacturing,

$$X^B(t) = L^B - L_n^B(t), \tag{9}$$

where L^B represents the economy's constant supply of homogeneous labor and $L_n^B(t)$ the amount of this pool of labor allocated to R&D.

At any moment of time, the market for final goods is assumed to be in equilibrium so that consumption $C^B(.)$ equals the flow of final output

$$Y^B(t) = C^B(t). \tag{10}$$

In the following analysis we assume that instantaneous utility is given by

$$U(t) = \log C^B(t). \tag{11}$$

Of course, one could work with a more general utility function. In fact, (11) is a limiting case of the widely used constant-elasticity-of-intertemporal-substitution utility function

$$U(t) = \frac{C(t)^{1-\theta} - 1}{1-\theta}$$

as $\theta \to 1$. For simplicity, we restrict ourselves to this limiting case.

Combining (7), (9), (10) as well as (11), and discounting by the time preference rate ρ we obtain the expression in the integral defining the objective function.

Let us next turn to equation (2) in the above optimal control problem. In the spirit of Romer, 1990, we employ a production function for developing blueprints for novel intermediates where the productivity of resources devoted to R&D is enhanced by the accumulated stock of knowledge capital. In the basic model with expanding product variety, a country's current stock of knowledge capital is simply equated with the number of intermediate goods invented in that country so far. A distinguishing feature of the leader-follower model is that the knowledge stock available in the follower country B at time t is assumed to consist of the sum of the knowledge accumulated in country B which is represented by the number of differentiated inputs developed so far domestically, $n^B(t)$, *and* a term comprising externally produced knowledge appropriated by country B. More specifically, a fraction $0 \leqslant \gamma\left(n^B\right) \leqslant 1$ of the knowledge stock produced in country A is absorbed into the knowledge stock of country B. Function $\gamma(n^B)$ represents the absorptive capacities (see Cohen and Levinthal, 1989) of the follower (determined by its capabilities but also by barriers to international communication or the extent of redundant knowledge which will not be targeted by the follower). For simplicity, in the present optimization problem we treat the absorptive capacities of the follower country as a parameter, γ. Finally, parameter a reflects productivity in R&D.

Equation (3) tells us that the autarkic leading country's stock of knowledge grows exponentially at the steady rate of innovation $\bar{g}^A > 0$. If the leading country evolves in its steady state, we know from Grossman and Helpman that its exponential rate of innovation is given by

$$\bar{g}^A = (1-\alpha)\frac{L^A}{a} - \alpha\rho > 0. \tag{12}$$

Equation (4) fixes initial conditions. Finally (see (5)), it is assumed that the follower country's R&D labor does not exhaust its total labor force and thus manufacturing activity does not vanish at any instant of time.

The analysis reported in this paper is restricted to the case

$$\bar{g}^A > L^B/a.$$

This inequality has a straightforward interpretation. If research productivity is identical in the two countries it says that the steady-state amount of labor allocated to R&D in the leading country exceeds the total labor force in the follower country. This suggests that - given uniform R&D productivity - the follower country is quite small relative to the leader. As shown in Hutschenreiter, Kaniovski and Kryazhimskii, 1995, the opposite inequality must hold for the follower country to be able to catch up with the leader in terms of knowledge stocks in the decentralized case.

At present, we have tentative results for the slightly relaxed constraint

$$\bar{g}^A > L^B/a - \rho/a$$

as well as for the opposite case

$$\bar{g}^A \leq L^B/a - \rho/a.$$

Clearly, the latter case fulfills the necessary condition which, according to Hutschenreiter, Kaniovski and Kryazhimskii, 1995, must be fulfilled for the follower country to be able to catch up with the leader in terms of its knowledge stock. In the present paper we will not deal with the last two cases.

3 Mathematical approach and main results

In this section we comment on the mathematical approach taken and report major results of the analysis. This analysis is carried out within the framework of mathematical optimal control theory (Pontryagin, et al., 1962). An important feature of the problem under consideration is that the goal functional is defined on an infinite time interval. In problems with infinite time horizons the application of the Pontryagin maximum principle, the key instrument in optimal control theory, is, in general, less efficient than in problems with finite time horizons. In particular, for the case of infinite time horizons the natural transversality conditions, providing as a rule essential information on the solutions, may not be valid (Halkin, 1974). Another important feature of the problem under consideration lies in the presence of a logarithmic singularity in the goal functional. This singularity poses additional difficulties and makes it impossible to apply the known mathematical results to this problem directly.

The analysis performed by Aseev, Hutschenreiter and Kryazhimskii, 2002, is based on the approximation approach to the investigation of optimal control problems with infinite time horizons developed recently in Aseev, Kryazhimskii and Tarasyev, 2001. This approach provides a possibility to establish existence results for problems with infinite time horizons and to derive the appropriate versions of the Pontryagin maximum principle which contain some extra conditions on the adjoint function and the behavior of the Hamiltonian at the infinity (in fact, this allows us, in some cases, to guarantee the validity of the additional transversality conditions at the infinity).

In the mathematical analysis, based on a suitable transformation of the original problem, the existence of an optimal solution of the optimal control problem (1) - (5) was shown. For a complete statement see Aseev, Hutschenreiter and Kryazhimskii, 2002.

To start with, let us define the ratio of knowledge stocks as

$$r(t) = \frac{n^B(t)}{n^A(t)}.$$

A major result of the mathematical analysis is that the unique optimal ratio of knowledge stocks approaches the positive constant

$$r^* = \gamma \frac{2\frac{L^B}{a}\left(\frac{1}{\alpha}-1\right) - \frac{1}{\alpha}\bar{g}^A - \rho + \left[\left(\frac{1}{\alpha}\bar{g}^A + \rho\right)^2 + 4\frac{L^B}{a}\left(\frac{1}{\alpha}-1\right)^2 \bar{g}^A\right]^{1/2}}{2\left(\left(\frac{1}{\alpha}-1\right)\left(\bar{g}^A - \frac{L^B}{a}\right) + \rho\right)}, \tag{13}$$

Furthermore, the control variable $L_n^B(t)$ approaches the positive constant

$$L_n^{B*} = \frac{a\bar{g}^A}{1+\frac{\gamma}{r^*}}. \tag{14}$$

Also, the adjoint variable $p(t)$ of the (redefined) optimal control problem examined approaches the positive constant

$$p^* = \frac{1}{\gamma\frac{L^B}{a} - \left(\bar{g}^A - \frac{L^B}{a}\right) r^*}.$$

This variable can be interpreted as the current value shadow price associated with the ratio of knowledge stocks.

4 A comparative analysis of the market and the optimal outcome

As noted, a major result of the mathematical analysis of the optimal control problem examined is that, under the present assumptions, there is an unique optimal asymptotic ratio of the stocks of knowledge of the two countries. This result implies that the knowledge stocks of the two countries grow at an identical exponential rate as time goes to infinity. Since the rate of innovation of the leading country is a parameter in our maximization problem, the asymptotic rate of innovation of the follower country equals that of the leading country.

This result was also found to hold asymptotically for a perfect-foresight equilibrium trajectory in the decentralized leader-follower model (Hutschenreiter, Kaniovski and Kryazhimskii, 1995). Thus we conclude that in terms of the asymptotic rate of innovation, social planning produces exactly the same result as the market mechanism. This unsettles a tenet which - as, e.g., pointed out by Aghion and Howitt, 1998 - is often, albeit wrongly attributed to endogenous growth theory. R&D-based endogenous growth models may well produce uniform long-run growth rates across countries.

However, it is possible that the long-run ratio of knowledge stocks associated with the market outcome differs from that in the optimal solution. In order to proceed towards a welfare analysis of the leader-follower model we will compare the asymptotically optimal amount of labor allocated to R&D, L_n^{B*}, to the long-run market allocation $L_{n\infty}^B = \lim_{t\to\infty} L_n^B(t)$. This is equivalent to a comparison of the optimal ratio of knowledge stocks, r^*, to the market outcome $r_\infty = \lim_{t\to\infty} r(t)$.

According to Hutschenreiter, Kaniovski and Kryazhimskii, 1995, in the market solution the asymptotic ratio of knowledge stocks is a simple function of the absorptive capacities of the follower and relative country size. Specifically, in the decentralized case, a perfect-foresight equilibrium trajectory was shown to be characterized by $n^B(t)$ growing to infinity, and the asymptotic ratio of knowledge stocks of the two countries, $r(t) = n^B(t)/n^A(t)$, approaching the positive constant

$$r_\infty = \frac{\gamma}{(L^A/L^B) - 1}. \tag{15}$$

Thus, e.g., in the case where country A is twice the size of country B, the relative knowledge stock of the latter simply equals its absorptive capacity.

In contrast, solving the above optimization problem we arrive at the considerably more complex expression (13) for the ratio of knowledge stocks. Thus, in general (except for particular parameter constellations), the asymptotic relative knowledge stocks derived from the two models are not the same. Indeed, we have proved (see Aseev, Hutschenreiter and Kryazhimskii, 2002) that the optimal asymptotic ratio of knowledge stocks exceeds the corresponding asymptotic ratio achieved by the market, i.e.

$$r^* > r_\infty.$$

If the the optimal ratio of knowledge stocks is higher than the respective market limit ratio then it follows that the market allocates too little labor to R&D. To illustrate this, let us divide both sides of equation (2) by $n^B(t)$. Using notation $g^B(t) = \dot{n}^B(t)/n^B(t)$ we get

$$g^B(t) = \frac{L_n^B(t)}{a}\left(1 + \frac{\gamma}{r(t)}\right).$$

Resolving this equation for $L_n^B(t)$ yields

$$L_n^B(t) = \frac{ag^B(t)}{1 + \frac{\gamma}{r(t)}}. \tag{16}$$

We have shown that $\bar{g}^A$ is the asymptotic rate of innovation of the leading and the follower country in both the market and the optimal solution. Fixing the rate of innovation at this value and passing to the limit, for the decentralized case, (16) becomes

$$L_{n\infty}^B = \frac{a\bar{g}^A}{1 + \frac{\gamma}{r_\infty}}. \tag{17}$$

In the case that, for a given rate of innovation, the asymptotic ratio of knowledge stocks is small, i.e. the leading country approaches a relatively large stock of knowledge, the follower will devote only little resources to its own R&D activities. The reason for this is that the productivity of the follower country's researchers is strongly boosted by knowledge absorbed from the leader so that it has to devote relatively little resources to own R&D in order to reach the leader's rate of innovation. On the other hand, if the knowledge stock the follower country achieves in the long run gets large relative to that of the leading country, its R&D labor input asymptotically approaches $a\bar{g}^A = \bar{L}_n^A$, i.e. the steady-state R&D labor input of the leading country, from below.

Since in the market outcome the asymptotic ratio of knowledge stocks is given by (15), and taking into account equation (17), the amount of labor allocated to R&D approaches

$$L_{n\infty}^B = \frac{L^B}{L^A}a\bar{g}^A. \tag{18}$$

It follows that in the long run, the shares of R&D employment in the total labor force are the same in both countries, namely $a\bar{g}^A/L^A$.

In contrast, for the optimal solution we have (14). Since we have established that that $r^* > r_\infty$ it follows immediately that

$$L_n^{B*} > L_{n\infty}^B,$$

i.e. the long-run market allocation of labor to R&D is less than optimal. Clearly, the limit optimal share of labor allocated to R&D in the follower country is higher than that in the market solution as well as that in the leading country.

To summarize, in the market solution of the leader-follower model, the long-run values of the pair $r(t)$, $L_n^B(t)$ consistent with the uniform asymptotic rate of innovation $\bar{g}^A$ is given by $r_\infty(t)$, $L_{n\infty}^B(t)$ defined by equations (15) and (18). For the optimal solution the corresponding pair $r^*(t)$, $L_n^{B*}(t)$ is defined by (13) and (14). This latter, optimal pair strictly dominates the first one.

5 Conclusions

For the leader-follower model examined, we conclude that, for a relatively small follower country, dynamic optimization (social planning) produces exactly the same asymptotic rate of innovation as the market, i.e. it does not improve the market outcome in these terms. This result is at odds with a tenet wrongly attributed to endogenous growth theory. Our example shows that R&D-based endogenous growth models may well produce uniform long-run growth rates across countries. However, we have shown that in the market solution the long-run allocation of labor resources to R&D in the follower country is less than optimal and, correspondingly, that the optimal ratio of knowledge stocks is larger than the corresponding ratio in the market solution. This implies that in the market solution the level of productivity of the follower country relative to that of the leader is less than optimal. Thus we arrive at the "Solowian" conclusion that optimal policy intervention does not have any effect on the long-run growth rate but will affect long-run relative knowledge stocks and relative levels of productivity.

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