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GENERALIZED ROBINSON AND MARSHALL-LERNER CONDITIONS WITH POSITIVE IMPORT CONTENT OF EXPORTS: THE CASE OF TWO COUNTRIES

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Abstract: The ROBINSON and MARSHALL-LERNER conditions are necessary and sufficient for a devaluation to be successful and are therefore the backbone of the elasticities approach to the balance of payments. These conditions are generalized by taking into account some actual features of modern foreign trade: positive import content of exports and the local content issue. In view of these effects, studied in a two country, two goods context, it can be concluded that a devaluation is likely to be less successful under given price elasticities. These results are of particular relevance for small open economies (SMOPECs)

1. Introduction

The so-called elasticities approach to the balance of payments relies on the MARSHALL-LERNER condition. According to this approach the effect of a devaluation depends on the price derivatives of the import demand and export supply functions for goods. In particular, it is argued that a devaluation will be successful if and only if the sum of the export and import demand elasticities exceeds one.

In a recent article [Breuss (1984)] it has been pointed out that the usual MARSHALL-LERNER condition, as well as the more general ROBINSON condition are not based on realistic assumptions of how exports are produced. These conditions for a

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normal reaction of the foreign balance (trade balance or current balance) to exchange rate changes were derived under the assumption that exports are produced only with domestic resources. However, exports, especially those produced in small countries, consist of a considerable amount of imported inputs. Modern foreign trade is characterized by yet another pattern of inccreasing importance. In the attempt to mitigate the import pressure of highly competitive countries such as Japan, some European countries and the United States try to persuade exporters in Japan to use in their products (e. g., cars) inputs from countries where they would like to export.

Both cases, the case of imported intermediate products in exports ("import content of exports") and that of "local content" issue ("export content of imports"), are treated in this paper in the tradition of the ROBINSON and MARSHALL-LERNER conditions. But it is the goal of this paper to derive more general elasticities conditions for a normal reaction of the foreign balance after a devaluation. The ROBINSON and the MARSHALL-LERNER conditions are included as a special case. This is applied to the two country, two goods case.

2. Imlications of Generalized Robinson and Marshall-Lerner Conditions with Positive Import Content of Exports in the Case of Two Countries

Although the ROBINSON and MARSHALL-LERNER conditions are conditions relating to the special case of two goods and therefore cannot apply to a world of many goods, these conditions are the backbone of the elasticities approach to the balance of payments. These conditions have a simple interpretation and are therefore attractive for economic policy. Furthermore, recent attempts [e. g., Dixit and Norman (1980, pp. 222-229)] to relate the monetary approach to the balance of payments and the neoclassical approach with its emphasis on price elasticities in a general equilibrium framework imply that the MARSHALL-LERNER condition must be satisfied for the impact effect of a devaluation to be positive.

In this paper some actual features of modern foreign trade are dealt with in the neoclassical tradition of quantity reactions due to price changes. In an earlier article [Breuss (1984)] this theme was touched upon but only the case of positive import content of exports in one country was considered.

The general case which is considered here assumes that two countries exhange two goods (exports and imports). In both countries imports consist of two parts: one part is used for domestic absorption (non-export-needed imports). This part of imports is a function of import prices. The other part is independent of import price changes but is in a fixed relationship to changes in exports. Due to the specific production technology, whenever exports are increased a fixed portion "v" of imports (intermediate products) also must be increased in order to be used as inputs for the production of exports. This factor "v" is called "import content of exports".

Figure 1 illustrates this symmetry in the case of two countries (with the labels "home" and "foreign") which exchange two goods (exports and imports).

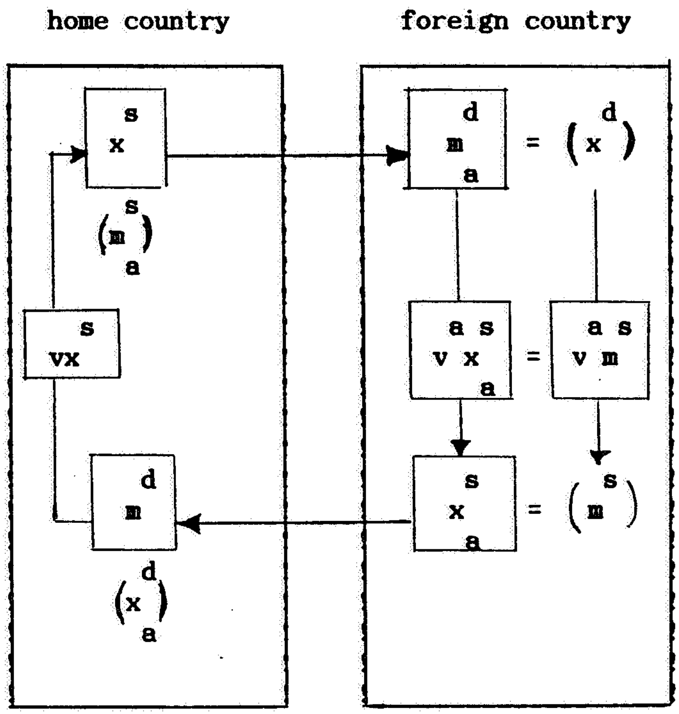

Figure 1: x^s, x^d is supply of and demand for real exports (home country);

x^d is equal to the import demand of the foreign country (m^d_a);

m^s, m^d is supply of and demand for real imports (home country);

m^s is equal to the export supply of the foreign country (x^s_a);

v, v^a is import content of exports in the home and foreign country.

From the point of view of the home country, one can define these reciprocal cases of impact of "positive import content of exports" in the following way:

Import demand in the home country is given by

$$m^{d} = (1-v)m'^{d}(p_{m}) + vx^{s}(p_{x}) \qquad (1)$$

When exports change, imports have to change by the factor "v": $\partial m^{d}/\partial x^{s} = v$. Two extreme cases occur if v = 0 and v = 1. In the former import demand is not related to export changes ($m^{d} = m'^{d}$). In the latter case exports are produced totally by means of import goods ($m^{d} = x^{s}$). Realistic figures from input-output tables suggest that "v" lies between 0.10 and 0.40. Smaller countries generally have a higher "v" than larger countries.

If one applies the same method to the foreign country, import demand is given by

$$m_{a}^{d} = (1-v^{a})m'^{d}_{a}(p^{a}_{m}) + v^{a}x^{s}_{a}(p^{a}_{x}) \qquad (2)$$

Because of the symmetry in the two country case (home country's exports are foreign country's imports and vice versa), from the point of view of the home country one can define (2) alternatively as

$$x^{d} = (1-v^{a})x'^{d}(p_{xa}) + v^{a}m^{s}(p\ ) \qquad (2a)$$

This equation says that export changes by the factor "v^{a}" are connected with import changes in the home country: $\partial x^{d} / \partial m^{s} = v^{a}$. Hence the factor "$v^{a}$" (which is the "import content of exports" in the foreign country) can also be interpreted as the "export content of imports" in the home country or as the "local content" factor.

The constellation when both "import contents of exports" are not (necessarily) equal and neither zero nor one, is called the GENERAL CASE ($0 < v < 1$; $0 < v^{a} < 1$; $v \neq v^{a}$).

The elasticities condition for the GENERAL CASE under the assumption of a balanced foreign balance and export as well as import price indices set equal to one (in the base period) is as follows [see Appendix, equation (A.18a) for a full derivation]:

$$\eta_{x'} \eta_{m'} [\varepsilon_{m} (1-v)(1-v^{a}) + \varepsilon_{x} (1-v)(1-v^{a}) + (1-v)(1-v^{a})] +$$

$$+ \varepsilon_{x} \eta_{x'} (1-v^{a}) v + \eta_{m'} \varepsilon_{m} (1-v) v^{a}$$

$$> \varepsilon_{x} \varepsilon_{m} [\eta_{x'} (1-v)(1-v^{a}) + \eta_{m'} (1-v)(1-v^{a}) + (1-vv^{a})] \qquad (3)$$

This rather messy expression can be reduced to the standard ROBINSON condition if and only if $v = v^{a} = 0$.

Under the assumption that $\varepsilon_{x}, \varepsilon_{m} \rightarrow \infty$ and after dividing both sides of equation (3) by $\varepsilon_{x} \varepsilon_{m}$ one gets

$$0 > \left[\eta_{x'}(1-v)(1-v^{a}) + \eta_{m'}(1-v)(1-v^{a}) + (1-vv^{a})\right] \quad (4)$$

If and only if $v = v^{a} = 0$ one is back at the original MARSHALL-LERNER condition.

The elasticities condition for the GENERAL CASE, given in equation (3), can be interpreted in the following way:

1. Like the original ROBINSON condition, it has the nice property of SYMMETRY. I. e. it gives the same results for $v > v^{a}$ and $v < v^{a}$. This implies that the elasticities condition is the same whether one considers a small county (high "v"), having trade relations with a large country (low "v^{a}"), or vice versa.

2. It can be fairly claimed that in recent decades the degree of interdependence of nations has increased due to foreign trade. Increased interdependence has lead to an increase in the "import content of exports" (v). This has consequences for the elasticities approach. The higher "v" and/or "v^{a}" the less probable satisfied are the elasticities conditions of equations (3) and (4). There is a clear positive relationship (or a trade-off) between "v" and/or "v^{a}" [or (1-v) and/or $(1-v^{a})$] and the elasticities. The higher the values of "v" and "v^{a}", the higher must be the values of the elasticities in order to fulfill the elasticities conditions for a devaluation to be successful.

In spite of the "symmetry" properties mentionned above, there is a clear lesson for the SMOPECs (small open economies). With a given "v^{a}" in the "foreign country" (or in the rest of the world), a small country (with a high "v") needs higher values of their elasticities than a large country (with a low "v") in order to get an improvement in the foreign balance after a devaluation.

3. Two extreme and unrealistic cases are thinkable. The first is, if $v = v^{a} = 0$, which brings one back to the familiar ROBONSON and

MARSHALL-LERNER conditions. The other case is, if $v = v^a = 1$. Because exports (imports) consist completely of imports (exports), a devaluation has no effect. This can be verified by inserting 0 and 1 respectively into the equations (3) and (4).

4. Sometimes the case of SMOPEC is treated by arguing that small countries are price takers and are therefore facing infinity export demand and import supply elasticities [e.g. Jarchow and Ruehmann (1982, p. 56)]. In particular it is assumed that $\eta_{x'} \longrightarrow -\infty$ and $\varepsilon_m \longrightarrow +\infty$. If one properly rearranges the terms of equation (A.18) of the Appendix so that on both sides the $\eta_{x'} \cdot \varepsilon_m$ term is removed and then all terms are divided by $-\eta_{x'}\varepsilon_m$ (taking into account $\eta_{x'} \longrightarrow -\infty$ and $\varepsilon_m \longrightarrow +\infty$) one gets the following SMOPEC elasticities condition for a devaluation to be successful:

$$\frac{x}{m}[(1-v^a) + \varepsilon_x (1-v)(1-v^a)] > [(1-v^a) + \frac{m}{m'}(1-v)(1-v^a)\eta_{m'}] \qquad (5)$$

If $x = m$, x/m and m/m' are cancelled out of equation (5).

But this case is not very interesting for several reasons. Firstly, whether ε_x and/or $\eta_{m'}$ are high or low, the elasticities condition of equation (5) is always satisfied for any values in the range of $0 < v < 1$ and/or $0 < v^a < 1$. Only in the limit, if $v = v^a = 1$, this condition becomes meaningless at all. Secondly, the infinity elasticities assumption for SMOPECs is probably exaggerated. The range of ε_m is difficult to evaluate because there are practically no estimates available for small countries. But $\eta_{x'}$ is, as can be seen from table 1 in the next section, definitely not infinite, even for small countries.

3. Some Empirical Results for the Generalized Marshall-Lerner Condition

In order to see how much the elasticities condition is influenced by the correction procedure for the positive "import content of exports", the generalized MARSHALL-LERNER condition of equation (4) is used. This exercise is done for some selected industrial countries for which comparable data on "η_x", "η_m" and "v" are available. The generalized ROBINSON condition, derived in equation (3), cannot be empirically tested as long as comparable data on "ε_x" and "ε_m" are available.

One sees from table 1 that "v" is generally higher for smaller countries (Netherlands, Belgium, Norway, Austria), and therefore the "correction factor" for the MARSHALL-LERNER condition is larger in the case of smaller countries.

In table 1 not only the correction for "v", the import content of export in the "home country" is made, but also for "v^a" of the "foreign country". This double correction is only a hypothetical one, made possible by assuming that "v^a" is equal for all countries. The "v^a", for illustrative purposes only, is calculated here as the average of the "v's" of the eight countries.

As one can see, nearly all chosen countries fulfill the original MARSHALL-LERNER condition before and after the correction procedure. But after the double correction procedure the elasticities condition for a normal reaction of the foreign balance after a devaluation is no longer satisfied for the United Kingdom and for the Netherlands. For Belgium the corrected condition is barely fulfilled. In the case of Austria, according to recent estimates for total trade (and only for total trade the correction factors (1-v) and $(1-v^a)$ are applicable), the MARSHALL-LERNER condition is satisfied neither before nor after the correction procedure. In contrast, the condition for SMOPEC, according to equation (5) would be fulfilled in the case of Austria if one takes into account recent estimates [e. g. Breuss (1983, p. 302)] for ε_x, which amount to +1.45.

But if one does not take the "best" point demand elasticities, quoted in table 1, it is questionable whether the generalized MARSHALL-LERNER condition is still satisfied for most of the countries in the sample, when additionally taking into account that the values for "v" probable have increased in the meantime.

Table 1

Import content of exports and MARSHALL-LERNER condition

Country	Import content of Exports	"Best" Point Long-run Elasticities of demand for (Total trade) [2]: Exports	Imports	MARSHALL-LERNER condition [a]: uncorrected	corrected for	
	v [1]	$\eta x'$	$\eta m'$	$v^a=0$, $v=0$	$v^a=0$, $v>0$	$v^a>0$, $v>0$
Germany (1965)	0.163	-1.11	-0.88	-0.99	-0.67	-0.32
France (1965)	0.134	-1.31	-1.08	-1.39	-1.07	-0.62
U.K. (1959)	0.192	-0.48	-0.65	-0.13	[b] +0.09	[b] +0.25
Italy (1965)	0.184	-0.93	-1.03	-0.96	-0.60	-0.27
Netherlands (1965)	0.316	-0.95	-0.68	-0.63	-0.12	[b] +0.07
Belgium (1965)	0.348	-1.02	-0.83	-0.85	-0.21	-0.01
Norway (1965)	0.253	-0.81	-1.19	-1.00	-0.49	-0.21
Austria (1976)	0.254	-0.93	-1.32	-1.25	-0.68	-0.35
Austria (1976) [3]	0.254	-0.53	-0.24	[b] +0.23	[b] +0.43	[b] +0.50

Sources:

[1] For Input-Output table 1959: Zienkowsky (1971); for Input-Output tables 1965: UN (1977); Austria Input-Output table 1976: Breuss and Skolka (1983).
[2] Stern, Francis and Schumacher (1976, p. 20).
[3] Breuss (1983, p. 283).

Comments:

[a] Refers to the sum of the right-hand side of equation (4). It is assumed that v^a is the average of the v's of the eight countries: $v^a = 0.231$
[b] MARSHALL-LERNER condition is not fulfilled.

4. Concluding Remarks

The ROBINSON and MARSHALL-LERNER conditions are necessary and sufficient for a devaluation to be successful in a two country, two goods case. It has been stressed in this article that some actual features of modern foreign trade can be captured in the framework of the elasticities approach to the balance of payments.

In particular, the consequences of fixed "import content of exports" as well as the "local content" issue have been studied. Although these elasticities conditions are a special case (two countries, two goods) in a world of many goods and many countries, the improvement of the MARSHALL-LERNER condition recommended in this article has policy relevance, especially for small open economies. It has been demonstrated that (abstracting from income-expenditure and wage-price effects) even the initial impact of a devaluation on the foreign balance is dampened when one takes into account that an increase in the volume of exports causes an increase in imports in both countries.

Appendix: Derivation of Generalized Robinson and Marshall-Lerner Conditions with Positive Import Content of Exports in a Two Country Case

Assumptions: Two countries (with the labels "home" and "foreign") and two goods (exports and imports). The derivation is done in domestic currency for the home country [For the standard derivation, see e. g. Jarchow and Ruehmann (1982, pp. 265-269)];

Export goods market

$$x^s(p_x) - x^d(p_{xa}) = 0 \qquad \text{(A.1)}$$

where:

$$p_{xa} = \frac{p_x}{w} \qquad \text{(A.2)}$$

x^s, x^d is supply of and demand for real exports; p_x, p_{xa} are export prices in domestic and foreign currencies; w is the exchange rate.

Import goods market

$$m^s(p_{ma}) - m^d(p_m) = 0 \qquad \text{(A.3)}$$

where: $$p_{ma} = \frac{p_m}{w} \qquad (A.4)$$

m^s, m^d is supply of and demand for real imports; p_m, p_{ma} are import prices in domestic and foreign currencies.

The case of positive import content of exports

Assume that in the home country the total volume of import demand (m^d) is statistically separable into two parts: one part is a function of import prices and is used for domestic absorption (non-export-needed imports); the other part is independent of import price changes but is a fixed proportion of exports. Let "v" be the proportionality factor.

The total volume of import demand in the home country is therefore defind as

$$m^d = (1-v)m'^d(p_m) + vx^s(p_x) \qquad (A.5)$$

and $$\frac{\partial m^d}{\partial x^s} = v; \qquad 0 < v < 1 \qquad (A.6)$$

where "v" is the "import content of exports" in the home country.
Equation (A.6) gives the change in imports necessary to accomodate the higher volume of exports [For a definition of "v", see also e. g. Georgakopoulos (1974, p. 199)].

The case of positive export content of imports (local content)

Assuming a similar production technology for the foreign country the reciprocal definition of import demand in the case of positive import content of exports in the foreign country is given by: $m^d_a = (1-v^a)m'^d_m(p^a) + v^s_a x^a(p_x)$. In the two country case import demand (export supply) in the foreign country is identical to export demand (import supply) in the home country ($m^d = x^d_a$ and $x^s_a = m^s$). One can therefore rewrite this relationship in terms of the home country as follows

$$x^d = (1-v^a)x'^d(p_{xa}) + v^a m^s(p_{ma}) \tag{A.7}$$

and
$$\frac{\partial x^d}{\partial m^s} = v^a ; \qquad 0 < v^a < 1 \tag{A.8}$$

where "v^a" is the "import content of exports" in the foreign country which is (in the two country case) equal the "export content of imports" ("local content") in the home country. Equation (A.8) tells how much changes of exports are linked to changes in imports.

Substituting equation (A.5) into into (A.3) one gets for the import goods market

$$m^{s}(p_{ma}) \quad - \quad [(1-v)m'^{d}(p_{m}) \; + \; vx^{s}(p_{x})] \quad = \quad 0 \qquad (A.3a)$$

Substituting equation (A.7) into (A.1) one gets for the export goods market

$$x^{s}(p_{x}) \quad - \quad [(1-v^{a})x'^{d}(p_{xa}) \; + \; v^{a}m^{s}(p_{ma})] \quad = \quad 0 \qquad (A.1a)$$

Foreign balance

$$p_{x}x^{s}(p_{x}) \quad - \quad p_{m}m^{d}(p_{m}) \quad = \quad B \qquad (A.9)$$

If one takes into consideration equation (A.3a) one obtains an alternative definition of the foreign balance for the home country

$$p_{x}x^{s}(p_{x}) \; - \; p_{m}(1-v)m'^{d}(p_{m}) \; - \; p_{m}vx^{s}(p_{x}) \; = \; B \qquad (A.9a)$$

<u>Differentiating with respect to w</u>

Export goods market (A.1a) under the condition (2)

$$\frac{dx^s}{dp_x}\,\frac{dp_x}{dw} - \frac{(1-v^a)dx'^d}{dp_{xa}}\left(\frac{w\frac{dp_x}{dw} - p_x}{w^2}\right) - \frac{v^a\,dm^s}{dp_{ma}}\left(\frac{w\frac{dp_m}{dw} - p_m}{w^2}\right) = 0 \qquad \text{(A.10)}$$

Import goods market (A.3a) under the condition (4)

$$\frac{dm^s}{dp_{ma}}\left(\frac{w\frac{dp_m}{dw} - p_m}{w^2}\right) - \frac{(1-v)dm'^d}{dp_m}\,\frac{dp_m}{dw} - \frac{vdx^s}{dp_x}\,\frac{dp_x}{dw} = 0 \qquad \text{(A.11)}$$

Foreign balance (A.9a), recalling that $(1-v)m'^d + vx^s = m^d$ and after proper rearrangement of terms

$$p_x\,\frac{dx^s}{dp_x}\,\frac{dp_x}{dw}\left(1-v\frac{p_m}{p_x}\right) + x^s\,\frac{dp_x}{dw} - (1-v)p_m\,\frac{dm'^d}{dp_m}\,\frac{dp_m}{dw} -$$

$$- m^d\,\frac{dp_m}{dw} = \frac{dB}{dw} \qquad \text{(A.12)}$$

If equations (A.10) to (A.12) are arranged in matrix form, one gets

$$
\begin{bmatrix}
\left(\frac{dx^s}{dp_x} - \frac{(1-v^a)dx^{d\prime}}{dp_{xa}} - \frac{1}{w}\right) & - \frac{v^a dm^s}{dp_{ma}} \frac{1}{w} & 0 \\
- \frac{v dx^s}{dp_x} & \left(\frac{dm^s}{dp_{ma}} \frac{1}{w} - \frac{(1-v)dm^{d\prime}}{dp_m}\right) & 0 \\
\left(p_x \frac{dx^s}{dp_x}\left(1 - \frac{p_m}{p_x}\right) + x^s\right) & -\left[(1-v)p_m \frac{dm^{d\prime}}{dp_m} + m^d\right] & -1
\end{bmatrix}
\begin{bmatrix}
\frac{dp_x}{dw} \\
\frac{dp_m}{dw} \\
\frac{dB}{dw}
\end{bmatrix}
=
\begin{bmatrix}
\left[- \frac{(1-v^a)dx^{d\prime}}{dp_{xa}} \frac{p_x}{w^2} - \frac{v^a dm^s}{dp_{ma}} \frac{p_m}{w^2}\right] \\
\frac{dm^s}{dp_{ma}} \frac{p_m}{w^2} \\
0
\end{bmatrix}
\qquad (A.13)
$$

Substituting the following price elasticities for

Export demand: $$\eta_{x'} = \frac{dx'^{d}}{dp_{xa}} \frac{p_{xa}}{x'^{d}}$$ (negative sign)

Export supply: $$\varepsilon_{x} = \frac{dx^{s}}{dp_{x}} \frac{p_{x}}{x^{s}}$$ (positive sign)

Import demand: $$\eta_{m'} = \frac{dm'^{d}}{dp_{m}} \frac{p_{m}}{m'^{d}}$$ (negative sign)

Import supply: $$\varepsilon_{m} = \frac{dm^{s}}{dp_{ma}} \frac{p_{ma}}{m^{s}}$$ (positive sign)

and taking into consideration the equilibrium conditions

$$x^{s} = x^{d} = x \qquad \text{and} \qquad m^{s} = m^{d} = m$$

as well as

$$p_{x} = p_{xa} w \qquad \text{and} \qquad p_{m} = p_{ma} w$$

matrix (A.13) can be rewritten in the following form

$$
\begin{bmatrix}
\frac{x'}{p_x}\left(\frac{x}{x'}\varepsilon_x - (1-v^a)\eta_{x'}\right) & -\frac{v^a m}{p_m}\varepsilon_m & 0 \\
-\frac{vx}{p_x}\varepsilon_x & \frac{m}{p_m}\left(\varepsilon_m - (1-v)\eta_{m'}\frac{m'}{m}\right) & 0 \\
x\left(1 + \left(1-v\frac{p_m}{p_x}\right)\varepsilon_x\right) & -m\left(1 + (1-v)\eta_{m'}\frac{m'}{m}\right) & -1
\end{bmatrix}
\begin{bmatrix}
\frac{dp_x}{dw} \\
\frac{dp_m}{dw} \\
\frac{dB}{dw}
\end{bmatrix}
=
\begin{bmatrix}
\left[-(1-v^a)\eta_{x'}\frac{x'}{w} - \varepsilon_m\frac{mv^a}{w}\right] \\
\varepsilon_m\frac{m}{w} \\
0
\end{bmatrix}
\qquad (A.14)
$$

The determinant of the coefficients of the left-hand side of (A.14) is

$$D = \frac{x'}{p_x}\left(\frac{x}{x'}\varepsilon_x - (1-v^a)\eta_{x'}\right)\frac{m'}{p_m}\left(\varepsilon_m - (1-v)\eta_{m'}\frac{m'}{m}\right) + \frac{vx}{p_x}\varepsilon_x\frac{v^a m}{p_m}\varepsilon_m$$

It is assumed that the matrix is non-singular, i. e. the determinant (D) is non-zero. This is the case if and only if $\varepsilon_x \neq \eta_{x'}$ (when $v^a = 0$) and $\varepsilon_m \neq \eta_{m'}$ (When $v = 0$).

Then, according to CRAMER's rule, (A.14) yields the following solutions, when a devaluation takes place.

For export prices

$$\frac{dp_x}{dw} = \frac{1}{D}\left[\left(-(1-v^a)\eta_{x'w}x' - \varepsilon_{mw}\frac{v^a m}{p_m}\right)\left(\varepsilon_m - (1-v)\eta_{m'm}m'\right)\right] -$$

$$-\frac{1}{D}\left[\varepsilon_{mw}\frac{v^a m}{p_m}\varepsilon_m\right] > 0 \qquad \text{(A.15)}$$

If and only if $v = v^a = 0$, one gets the simple standard result

$$\frac{dp_x}{dw} = - \frac{\eta_x \, p_x}{(\varepsilon_x - \eta_x)\, w} > 0 \qquad \text{(A.15a)}$$

For import prices

$$\frac{dp_m}{dw} = - \frac{1}{D} \left[\frac{x'}{p_x} \left(\frac{x}{x'} \varepsilon_x - (1-v^a)\eta_{x'} \right) \varepsilon_m \frac{m}{w} \right] +$$

$$+ \frac{1}{D} \left[\frac{vx}{p_x} \varepsilon_x (1-v^a) \eta_{x'} \frac{x'}{w} + \frac{vx}{p_x} \varepsilon_x \varepsilon_m \frac{m v^a}{w} \right] > 0 \qquad \text{(A.16)}$$

If and only if $v = v^a = 0$ one gets the standard result

$$\frac{dp_m}{dw} = \frac{\varepsilon_m \, p_m}{(\varepsilon_m - \eta_m)\, w} > 0 \qquad \text{(A.16a)}$$

For the foreign balance the solution of matrix (A.14) is a messy one. After multiplying out and rearranging in order to get export and import related terms (taking into consideration that $X = p_x x$ and $M = p_m m$) one obtains the condition for an improvement (deterioration) of the foreign balance after a devaluation ($dw > 0$):

$$\frac{dB}{dw} \gtreqless 0; \text{ if } \frac{X}{D}\left[\varepsilon_m \eta_{x'} (1-v^a) m x' + \varepsilon_m \varepsilon_x \eta_{x'} \left(1-v\frac{p_m}{p_x}\right)(1-v^a) m x'\right] -$$

$$- \frac{X}{D}\left[\eta_{m'} \eta_{x'} (1-v)(1-v^a) m' x' + \eta_{m'} \varepsilon_m (1-v) v^a m' m\right] -$$

$$- \frac{X}{D}\left[\eta_{m'} \eta_{x'} \varepsilon_x (1-v)\left(1-v\frac{p_m}{p_x}\right)(1-v^a) m' x'\right] -$$

$$- \frac{X}{D}\left[\varepsilon_x \varepsilon_m \eta_{m'} (1-v)\left(1-v\frac{p_m}{p_x}\right) v^a m' m\right] \gtreqless$$

$$\gtreqless \frac{M}{D}\left[\varepsilon_x \eta_{x'} (1-v^a) v x x' + \varepsilon_x \varepsilon_m v v^a m x\right] +$$

$$+ \frac{M}{D}\left[\varepsilon_x \eta_{m'} \eta_{x'} (1-v)(1-v^a)\frac{v m' x x'}{m} + \varepsilon_x \varepsilon_m \eta_{m'} (1-v)\frac{v v^a x m'}{m}\right] +$$

$$+ \frac{M}{D}\left[-\varepsilon_m \varepsilon_x m x - \varepsilon_m \varepsilon_x \eta_{m'} (1-v) m' x + \varepsilon_m \eta_{x'} (1-v^a) m x'\right] +$$

$$+ \frac{M}{D}\left[\varepsilon_m \eta_{m'} \eta_{x'} (1-v)(1-v^a) m' x'\right] \qquad \text{(A.17)}$$

This messy expression reduces to the standard ROBINSON condition if and only if $v = v^a = 0$

$$\frac{dB}{dw} \gtreqless 0; \text{ if } \quad X \frac{\eta_x(1 + \varepsilon_x)}{-(\varepsilon_x - \eta_x)} \gtreqless M \frac{\varepsilon_m(1 + \eta_m)}{(\varepsilon_m - \eta_m)} \qquad \text{(A.17a)}$$

By choosing units of measurement such that $p_x = p_m = 1$ (in the base period) and after rewriting (A.17), one obtains the condition for a normal reaction of the foreign balance after a devaluation for the GENERAL CASE ($0 < v < 1$; $0 < v^a < 1$; $v \neq v^a$):

$$\eta_{x'}\eta_{m'}\left[\varepsilon_m(1-v)(1-v^a) + \frac{x}{m}\varepsilon_x(1-v)(1-v^a) + \frac{x}{m}(1-v)(1-v^a)\right] +$$

$$+ \varepsilon_m\eta_{x'}(1-v^a)\left[\frac{m}{m'} - \frac{xm}{mm'}\right] + \varepsilon_x\eta_{x'}\left[(1-v^a)\frac{vx}{m'}\right] + \eta_{m'}\varepsilon_m\left[\frac{xm}{mx'}(1-v)v^a\right]$$

$$> \varepsilon_x\varepsilon_m\left[\frac{xm}{mm'}\eta_{x'}(1-v)(1-v^a) + \frac{x}{x'}\eta_{m'}(1-v)(1-v^a) + \frac{mx}{m'x'}(1 - vv^a)\right]$$

(A.18)

This expression can be further simplified by assuming that the foreign balance (in both countries) is in equilibrium. Recalling the definition of the foreign balance (if $p_x = p_m = 1$) for the home country: $B = x - m =$

= x − [(1−v)m' + vx]; if B = 0, x = m and x = m'; and for the foreign country:
$B^a = x^a - m^a = m - [(1-v^a)x' + v^a m]$; if $B^a = 0$, $x^a = m^a$ and $x' = m$ which also implies that m = m' and x = x'.
Then one obtains the simplified expression for the GENERAL CASE

$$\eta_{x'}\eta_{m'}[\varepsilon_m (1-v)(1-v^a) + \varepsilon_x (1-v)(1-v^a) + (1-v)(1-v^a)] +$$

$$+ \varepsilon_x \eta_{x'} (1-v^a)v + \eta_{m'} \varepsilon_m (1-v)v^a$$

$$> \varepsilon_x \varepsilon_m [\eta_{x'} (1-v)(1-v^a) + \eta_{m'} (1-v)(1-v^a) + (1 - vv^a)] \qquad \text{(A.18a)}$$

This condition reduces to the original ROBINSON condition if and only if $v = v^a = 0$

$$\eta_x \eta_m [\varepsilon_m + \varepsilon_x + 1] > \varepsilon_x \varepsilon_m [\eta_x + \eta_m + 1] \qquad \text{(A.18b)}$$

Additionally, one gets the original MARSHALL-LERNER condition if one assumes that $\varepsilon_x, \varepsilon_m \longrightarrow +\infty$ and when equation (A.18a) is divided on both sides by $\varepsilon_x \varepsilon_m$

$$0 > [\eta_x + \eta_m + 1] \qquad \text{(A.18c)}$$

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