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Product differentiation and competitive selection*

Serguei Kaniovski[†]

Abstract

By studying the effect of different patterns of demand in an evolutionary selection model this paper shows how product differentiation reduces competitive selection pressure and thus increases the chances for the coexistence of firms. With the example of a duopoly it shows that: (1) a monopoly is the likely outcome of competition in homogeneous products, (2) although product differentiation does not preclude a monopoly it greatly improves the chances for the stable coexistence of firms in the long run, and (3) the more differentiated the products, the more stable the duopoly.

JEL Codes: B52, L11

Key Words: product differentiation, selection, market niche

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1 Introduction

The conclusion commonly drawn in industrial organizations literature that competition in differentiated products is softer than competition in homogeneous products has a long intellectual lineage dating back to the contributions of Hotelling (1929) and Chamberlin (1933). At the inception of what became known as the structure-conduct-performance paradigm, Bain (1956, ch. 4) remarks on the dual effect of product differentiation, first, in reducing competition among the established firms and, second, as a barrier to entry.[1] The first of the two effects can be illustrated on the example of the classic Cournot and Bertrand models. Product differentiation increases firms' profits in a Cournot equilibrium, while it also resolves the Bertrand paradox as profits diverge from zero (Martin (1993, ch. 2)). It is the idea of product differentiation as an entry barrier, however, that has received an extended theoretical development, starting with the preemption theory of Eaton and Lipsey (1979) and Eaton and Lipsey (1980), and culminating in the endogenous sunk cost theory of Sutton (1991). While offering market niches for potential entrants, product differentiation achieved by advertising and product R&D can impede entry. Less actual competition coupled with higher entry barriers render the effect of product differentiation on market concentration ambiguous.

This paper shows at least the first of the two conclusions - that product differentiation eases competition among the established firms - to be true also for evolutionary models in the tradition of Nelson and Winter (1982). Product differentiation reduces selection pressure emanating from the product market and thus increases the chances for the coexistence of heterogeneous firms. This is shown on the example of a duopoly but will apply also to markets with more than two firms. Thus, by extending the homogeneous-product setting characteristic of a class of evolutionary models in the tradition of Nelson and Winter, this paper contributes to the discussion of the implications of more realistic demand structures in these and other evolutionary models.[2]

By means of product differentiation firms can partially insulate their own market segment by reducing its sensitivity to competitive moves. In their analysis of Schumpeterian competition, Nelson and Winter

[1]For a historical perspective and a survey of models see Beath and Katsoulacos (1991).

[2]Examples range from the early modeling efforts by Nelson and Winter (1982) and Winter (1984) to more complex models by Jonard and Yildizoglu (1998), Winter, Kaniovski, and Dosi (2000), and Winter, Kaniovski, and Dosi (2003). Surveys of evolutionary modeling in economics are available by Silverberg (1997) and Kwasnicki (2002).

(1982) emphasize the role of investment strategy in generating and limiting market concentration. The mechanism at play is that of selection. By expanding productive capacity, more efficient or otherwise more profitable firms can depress the price enough to squeeze their competitions out of the market. Nelson and Winter conclude that aggressive investment strategies are *ceteris paribus* conducive to higher concentration.[3] This paper shows that the impact of investment strategy on market concentration profoundly depends on the underlying demand structure.

1.1 Product differentiation and the market niche

Even a casual observation of entrepreneurial activity suggests that firms seek competitive advantage simultaneously in several domains. Given the variety in firm characteristics and in the terminology of the evolutionary perspective, market competition is a process of selection in which the firm, as the unit of selection, possesses multiple characteristics of selective significance (Metcalfe (1998)). It is the interaction between multiple characteristics of selective significance together with the interaction with the embedding environment that determine the performance of the firm and the market as a whole.

Put in this perspective, the nature of model solutions underscores an important principle of market competition succinctly put by Metcalfe: "when firms differ in more than one characteristic then competition does not select in general the most efficient firm nor does it always result in increases in the average efficiency with which resources are utilized" (Metcalfe (2002), p. 1). Success or failure is defined by the actual rather than conceivable competition, and *actual competition can leave room for inefficiency in some domains of entrepreneurial activity, which may or may not be compensated by efficiency in other domains.* This preservation of inefficiency may be the principal cause for the heterogeneity of entrepreneurial forms and behaviors actually observed. It is in this sense that the common wisdom of product differentiation creating *market niches* finds its validation.

Among the multitude of factors conducive to entrepreneurial success, the degree of product differentiation plays an important and sometimes even decisive role. If accepted in principle, the underlying

[3]The *ceteris paribus* proviso is significant, as Nelson and Winter's and most subsequent modeling efforts in their tradition in fact focus on the dynamic interplay of innovation and market structure, and different patters of innovative activity affect market structure differently (Geroski (1994); Audretsch (1995); Cohen (1995)). This innovation and market structure nexus is a cornerstone to the Schumpeterian tradition by which the evolutionary perspective in economics is, then as well as now, inspired (Dosi and Nelson (1994); Nelson (1995); Nelson and Winter (2002)).

selection mechanism must apply also to the infinitely more complex Schumpeterian environments, in which entry and exit, process and product innovation and advertising keep every niche constantly in flux.

1.2 The modeling approach

To isolate the effect of product differentiation on the strength of selection, two simplifying assumptions are made. First and foremost, the model sets innovation and its consequences for market structure aside. Consequences that we know are likely to be substantial, as innovation may change the firm's operating environment as well as its fitness in that environment. Second, to separate the effect of product differentiation from that of demand growth, a sales market with a fixed carrying capacity is assumed. This assumption is consistent with a satiable sales market in which movements along a demand function may occur, but shifts of the demand function are relatively unimportant. Consequently, market capacity is gradually exhausted as firms expand.

Product differentiation is introduced through the familiar concepts of own- and cross-elasticities of demand in a way consistent with the definition of strategic substitutes (complements) by Bulow, Geanakoplos, and Klemperer (1985). This approach is simple and at the same time sufficiently general to allow a direct comparison of different demand patterns. Its simplicity lies in contrast with the existing evolutionary models that feature differentiated products. For example, Silverberg, Dosi, and Orsenigo (1988) differentiate products by their price and delivery time, Windrum and Birchenhall (1998) model product differentiation using a utility function, whereas Kwasnicki and Kwasnicka (1992) and Metcalfe (1998, pp. 77-84) resort to an abstract competitiveness function in which competitiveness of a product is measured relative to the average competitiveness of all products in the market. With few exceptions, all the above models combine several economic phenomena that stochastically interact in an involved or inextricable manner. The appealing realism comes at an obvious price. Even if the relevant phenomenon is explicitly modeled, its effect is easily obscured by the resulting complexity, while the variety of modeling approaches frustrates comparisons between the models.

Patterns of demand relations are embedded into a dynamic model of capital accumulation borrowed from Winter, Kaniovski, and Dosi (2003), less stochastic entry and exit. Exclusion of stochastic entry renders the model deterministic and thus permits an analytic inquiry into the properties of the selection process implied by the underlying dynamical system. This dynamic element is fairly representative for evolutionary models in the tradition of Nelson and Winter (1982).

The remainder of the article is structured as follows. After introducing the basic model, Section 2 compares the stability of a duopoly selling homogeneous products to the stability of a duopoly selling imperfect substitutes. Section 3 discusses possible extensions to an arbitrary number of firms, more varied demand patterns and investment behaviors. Section 4 offers concluding remarks on the relative strengths of selective pressures in the two market environments.

2 The model

Substitutability between the two products is captured by different functional forms of a inverse demand function h, which relates market output to the market price. For homogeneous products the inverse demand function is common to both firms and takes the form $h : \mathbb{R}_+ \to \mathbb{R}_+$. In the case of differentiated products a pair of firm-specific linear inverse demand functions h_1, $h_2 : \mathbb{R}^2_+ \to \mathbb{R}_+$ is assumed.[4]

Patterns of demand relations are embedded into a dynamic model of capital accumulation defined by an investment rule, a law of evolution of the net capital stock and an AK production technology. At time t, Firm j produces output $q_j(t) \geq 0$ and pays wage in efficiency units $w_j > 0$. At the market-clearing price, the firm's cash flow is given by $(h_j(t) - w_j)q_j(t)$. A part of cash flow is used to purchase $y_j(t)$ units of capital goods at $v_j > 0$ per unit, for an investment outlay

$$v_j y_j(t) = \lambda_j [h_j(t) - w_j]^+ q_j(t) \ , \tag{1}$$

where $\lambda_j \in (0, 1]$ is the firm's propensity to invest.[5] Capital $k_j(t)$ is accumulated according to a continuous-time version of the perpetual inventory method. Given an initial capital endowment $k_j(0) > 0$

[4]Function h is strictly decreasing. Functions h_1, h_2 are strictly decreasing in the own output. All three are differentiable.
[5]Function $[x]^+ \equiv \max[x, 0]$ ensures that investment is nonnegative.

and a depreciation rate $\rho \in (0, 1]$ the net change in capital stock follows

$$\dot{k}_j(t) = y_j(t) - \rho k_j(t) \ . \tag{2}$$

Firm j's productive efficiency is given by capital per unit of output a_j, so that

$$\dot{q}_j(t) = a_j \dot{k}_j(t) \ . \tag{3}$$

Together equations (1) to (3) lead to the following growth equation:[6]

$$\dot{q}_j(t) = \left\{ \frac{\lambda_j a_j}{v_j} [h_j(t) - w_j]^+ - \rho \right\} q_j(t) \ . \tag{4}$$

The formal part of the paper studies the stability of a system defined by a pair of growth equations, holding propensities to invest constant. This is a model of a duopoly in which the firms maximize turnover. More flexible investment strategies are discussed in Section 3.1.

2.1 Homogeneous products

Growth equation (4) implies the existence of a unique total output Q_j^* such that $\lambda_j a_j v_j^{-1} [h(Q_j^*) - w_j]^+ = \rho$ at which the firm's productive capacity stagnates. The significance of Q_1^* and Q_2^* is best conveyed in terms of the market prices $h(Q_1^*)$ and $h(Q_2^*)$, the *break even* prices. Let $Q_1^* > Q_2^*$, so that Firm 1 has the lower break even price. A firm can grow as long as its cash flow is high enough for the net investment to be positive. While $Q(t) < Q_2^*$, both firms grow, depressing the market price to accommodate the higher total output. At $Q(t) = Q_2^*$ Firm 2's output stagnates, while Firm 1 continues to grow until $Q(t) = Q_1^*$. Outputs beyond Q_1^* can neither be reached, nor sustained if chosen initially. Between Q_2^* and Q_1^*, Firm

[6] The differences between this specification and a continuous-time analogue of the deterministic part in Winter, Kaniovski, and Dosi (2003) are superficial. Here efficiency is defined by the capital per unit of output rather than its reciprocal, and w is viewed as wages rather than total marginal or average costs. The latter should properly include investment outlays. Also note that a common ρ entails no loss of generality.

2's cash flow dries up, forcing it to exit the market. Firm 1 eventually becomes a monopoly with

$$h(Q_1^*) = \frac{\rho v_1}{\lambda_1 a_1} + w_1 \ . \tag{5}$$

The equilibrium market price is a weighted sum of factor costs in efficiency units, with the weight of the perishable input factor - labor - being equal to unity. Note that while maximizing the quantity sold, a constant propensity to invest unrealistically implies zero monopoly profits in the long run. Section 3.1 shows that more sophisticated investment strategies need not have this property.

It will be useful for the following discussion to describe the above dynamics in more general terms. Let $Q^* = \max(Q_1^*, Q_2^*)$ and $Q_* = \min(Q_1^*, Q_2^*)$, where

$$Q_1^* = h^{-1}\left(\frac{\rho v_1}{\lambda_1 a_1} + w_1\right) \quad \text{and} \quad Q_2^* = h^{-1}\left(\frac{\rho v_2}{\lambda_2 a_2} + w_2\right) . \tag{6}$$

Since

$$\dot{q}_1(t),\ \dot{q}_2(t) < 0 \quad \text{for} \quad Q(t) > Q^* \ ; \tag{7}$$

$$\dot{q}_1(t),\ \dot{q}_2(t) > 0 \quad \text{for} \quad Q(t) < Q_* \ ; \tag{8}$$

$$\text{opposite signs} \quad \text{for} \quad Q(t) \in (Q_*, Q^*) \ , \tag{9}$$

the firms' growth paths tend to the shaded set in Figure 1. This set is delineated by a pair of isoclines L_1 and L_2. An isocline L_j is the locus of all pairs (q_1, q_2) such that $q_1 + q_2 = Q_j^*$, or

$$L_j = \left\{(q_1, q_2) \in \mathbb{R}_+^2 \quad \text{such that} \quad \frac{\lambda_j a_j}{v_j}(h(Q) - w_j) = \rho\right\} . \tag{10}$$

In the example above: $Q^* = Q_1^*$ and $Q_* = Q_2^*$. For positive initial outputs $(Q_1^*, 0)$ is a *globally asymptotically stable* (GAS) equilibrium.[7] Similarly, if Firm 2 has the lower break even price, then it would emerge the monopolist. We arrive at the proposition that monopoly is inevitable unless the break even

[7] The proof is obtained from the stability of the Jacobian evaluated at $(0, 0)$, $(Q_1^*, 0)$ and $(0, Q_2^*)$.

prices coincide.

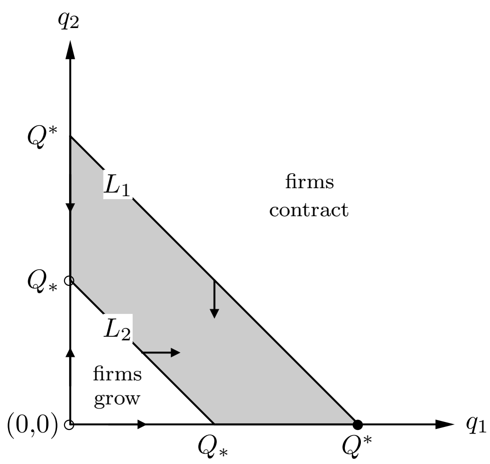
Proposition 1: *In the case of homogeneous products the condition for a stable duopoly is knife-edge:*

$$\frac{\rho v_1}{\lambda_1 a_1} + w_1 = \frac{\rho v_2}{\lambda_2 a_2} + w_2 \ . \tag{11}$$

When satisfied, it leads to a solution in which the equilibrium market shares are positive for positive initial capital endowments (Appendix A.1). The condition is satisfied when the two firms are identical.

Absent any theoretical basis for meeting this remarkably precise requirement, a prolonged coexistence of firms competing in homogeneous products is unlikely unless the firms are identical.

Figure 1: **Homogeneous products**



The encircled points are stationary for $Q_1^* > Q_2^*$. The filled point is globally asymptotically stable

2.2 Imperfect substitutes

Let us now see what happens when imperfect substitution is introduced. The new model is obtained by replacing the general inverse demand function by a pair of firm-specific linear inverse demand functions

$$h_1(q_1, q_2) \quad = \quad A_1 - B_1 q_1 - C_1 q_2 \ ; \tag{12}$$

$$h_2(q_1, q_2) \quad = \quad A_2 - B_2 q_2 - C_2 q_1 \ , \tag{13}$$

where all parameters are positive and $B_1 > C_1$, $B_2 > C_2$. The system of growth equations now becomes

$$\dot{q}_1(t) = \left\{\frac{\lambda_1 a_1}{v_1}[A_1 - B_1 q_1(t) - C_1 q_2(t) - w_1]^+ - \rho\right\} q_1(t) \; ; \tag{14}$$

$$\dot{q}_2(t) = \left\{\frac{\lambda_2 a_2}{v_2}[A_2 - B_2 q_2(t) - C_2 q_1(t) - w_2]^+ - \rho\right\} q_2(t) \; , \tag{15}$$

where all parameters unrelated to demand are as in the case of homogeneous products.

Again, a firm earns sufficient cash flow to grow as long as the market price of its product exceeds the firm's break even price. Isoclines are the loci of all pairs (q_1, q_2) that support a firm's break even price

$$L_1 = \left\{(q_1, q_2) \in \mathbb{R}^2_+ \quad \text{such that} \quad \frac{\lambda_1 a_1}{v_1}(A_1 - B_1 q_1 - C_1 q_2 - w_1) = \rho\right\} \; ; \tag{16}$$

$$L_2 = \left\{(q_1, q_2) \in \mathbb{R}^2_+ \quad \text{such that} \quad \frac{\lambda_2 a_2}{v_2}(A_2 - B_2 q_2 - C_2 q_1 - w_2) = \rho\right\} \; . \tag{17}$$

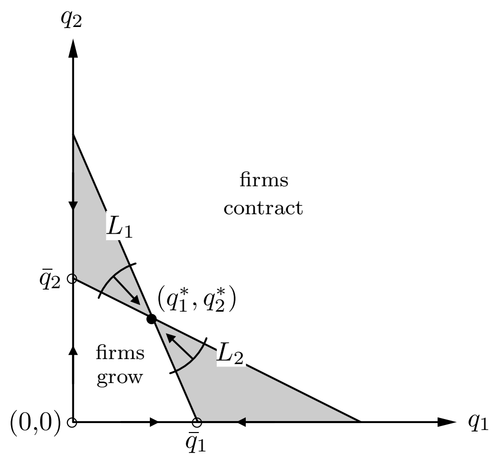
In $\mathbb{R}^2_+$, the isoclines either intersect, or do not intersect, or coincide. Their relative position defines the number, arrangement and stability properties of the equilibria.

Let the isoclines intersect in $\mathbb{R}^2_+$ (Figure 2). For combinations of individual outputs lying below the isoclines each firm earns sufficient cash flow to grow. The shaded region to the left represents all combinations of individual outputs that yield a market price of Product 1 below the break even price of Firm 1, and a market price of Product 2 above the break even price of Firm 2. In this region Firm 1 grows, while Firm 2 contracts. The opposite holds true of all combinations of individual outputs belonging to the shaded region to the right. Finally, for all combinations of individual outputs lying above the isoclines market prices are such that both firms contract. This situation occurs only when both firms start with outputs that cannot be sustained at market entry. There are now four stationary points, but only the intersection is GAS. Independently of initial (positive) capital endowments, individual outputs thus converge to the vector

$$\left(\frac{[A_1 - w_1 - \frac{\rho v_1}{\lambda_1 a_1}]B_2 - [A_2 - w_2 - \frac{\rho v_2}{\lambda_2 a_2}]C_1}{B_1 B_2 - C_1 C_2}, \frac{[A_2 - w_2 - \frac{\rho v_2}{\lambda_2 a_2}]B_1 - [A_1 - w_1 - \frac{\rho v_1}{\lambda_1 a_1}]C_2}{B_1 B_2 - C_1 C_2}\right) . \tag{18}$$

Recall that the degree of product differentiation is given by $B_1 - C_1$ and $B_2 - C_2$. Whether and how it

Figure 2: **Imperfect substitutes**



The encircled points are stationary for $B_1 > C_1$, $B_2 > C_2$. The filled point is globally asymptotically stable

affects the *feasibility* of the above stable solution is established by the following proposition:

Proposition 2: *The more differentiated the products, the more stable the duopoly.*

Proof: Since the denominator in (18) is positive, an intersection in the interior of $\mathbb{R}^2_+$ is feasible iff

$$\left(A_1 - w_1 - \frac{\rho v_1}{\lambda_1 a_1}\right) B_2 - \left(A_2 - w_2 - \frac{\rho v_2}{\lambda_2 a_2}\right) C_1 \quad > \quad 0\ , \tag{19}$$

$$\left(A_2 - w_2 - \frac{\rho v_2}{\lambda_2 a_2}\right) B_1 - \left(A_1 - w_1 - \frac{\rho v_1}{\lambda_1 a_1}\right) C_2 \quad > \quad 0\ , \tag{20}$$

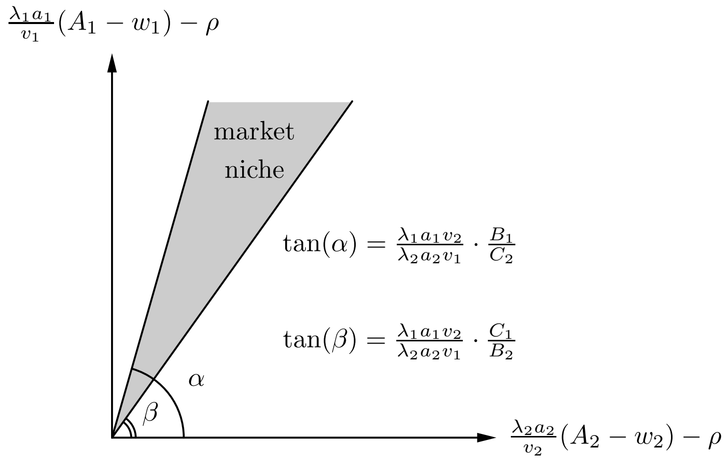
subject to the mild condition $A_j - w_j - \rho v_j/(\lambda_j a_j) > 0$ that supposes a sufficient carrying capacity in the market. In the space spanned by $\lambda_1 a_1 v_1^{-1}(A_1 - w_1) - \rho$ and $\lambda_2 a_2 v_2^{-1}(A_2 - w_2) - \rho$, the solution to (19)-(20) is given by the shaded area delimited by two lines with slopes given by

$$\frac{\lambda_1 a_1 v_2}{\lambda_2 a_2 v_1} \cdot \frac{B_1}{C_2} > \frac{\lambda_1 a_1 v_2}{\lambda_2 a_2 v_1} \cdot \frac{C_1}{B_2} > 0\ . \tag{21}$$

The geometry of Figure 3 reveals that the larger $B_1 - C_1$ and $B_2 - C_2$, the more likely the isoclines are to intersect in the interior of $\mathbb{R}^2_+$, the more stable the duopoly.

QED

Figure 3: **Stability with imperfect substitutes**



All other things being equal, the larger $B_1 - C_1$ and $B_2 - C_2$, the larger the set of parameters that supports stable duopoly

Figure 3 highlights a significant difference between the two stability requirements. When the products are homogeneous the condition is *knife-edge*. With imperfect substitution, however, the analogous condition is considerably less severe and can more appropriately be described as delimiting a *market niche*.[8] The more differentiated the products, the larger the niche. While product differentiation can considerably soften competition, it will not completely negate it. A monopoly is still feasible if one firm is vastly superior to the other in terms of efficiency, or costs of production, or if the degree of product differentiation is insufficient.[9] This being the case, the firm's outputs undergo essentially the same dynamics as with homogeneous products. What the model therefore suggests is that entrepreneurial success depends on a whole range of factors, rather than on a single factor, but also that under constant demand conditions the degree of product differentiation seems to be one of the more important factors.

The difference between a *knife-edge* and a *market niche* equilibrium can also be shown in terms of probabilities of their occurrence. If two firms with random productivities and production costs enter the market, as they often do in stochastic models, then the probability of duopoly being stable is zero when the products are homogeneous and positive when the products are differentiated.

[8] The *knife-edge* versus *niche* characterization is common in the international economics (Glenn and Fudenberg (2003)).
[9] In Figure 2 this will occur when the isoclines intersect outside or the boundary of $\mathbb{R}^2_+$.

I conclude this section by a remark on nonlinearity of demand functions. Strictly decreasing but nonlinear demand functions do not alter the nature of model solutions. For a pair of general but differentiable demand functions the system (14)-(15) is *competitive* in the sense of Hofbauer and Sigmund (1998, p. 28-29). A competitive system is one in which both entities (in our case, firms) are harmed by interaction and a cooperative system is one in which both entities benefit from interaction. Markets in which all products are mutual substitutes or mutual complements provide examples. Either type of system fulfills the Bendixson condition for the non-existence of periodic solutions, meaning that individual outputs must converge either to infinity or to a stationary point. As monotonicity of demand functions eliminates the former possibility, they must converge to a stationary point. Such points either lie on the boundary (monopoly) or in the interior of the phase space (stable duopoly). However, nonlinear demand functions can lead to multiple stable equilibria when the resulting nonlinear isoclines have multiple interior intersections, each necessarily being a stable equilibrium.

3 Discussion and extensions

3.1 The role of investment strategy

Constant propensities to invest describe the situation when firms maximize turnover. Among conceivable alternative behavioral rules, one that keys λ to the ratio of price to variable costs per unit of output, or the markup, warrants special attention. Initially suggested in Nelson and Winter (1982), this rule mimics myopic profit-maximizing behavior and can lead to less aggressive investment behavior thus improving stability in either market environment. To see why, consider once again the situation in which Firm 1 monopolizes the market. Its current profit is given by the difference between cash flow and investment outlays

$$\pi_1(t) = (1 - \lambda_1)[h(Q(t)) - w_1]^+ q_1(t) \ ,$$

is nonnegative as $h(Q_1^*) > w_1$, but not necessarily monotone in t. Being a myopic profit-maximizer, Firm 1 would not expand beyond the output at which $\dot{\pi}_1(t) = 0$. Given sufficiently small initial capacities this will be a maximum rather than a minimum of $\pi_1(t)$, perhaps a local one (hence myopic). Such an output level may or may not be reached. However, if reached while the market price is still above the break even price of Firm 2, then the duopoly output would converge to Q_2^* rather than Q_1^*, and both firms would survive. Myopic profit maximization can lead to more moderate investment behavior and thus make otherwise very unstable homogeneous product duopoly more stable. By the same token it could also improve stability in the case of imperfect substitutes.

While myopic profit maximization can soften competition, rational expectations can only have the opposite effect. Indeed, equation (11) implies that the higher the propensity to invest, the more likely the firm will survive. With homogeneous products, Firm 1's strategy under rational expectations will be to permanently invest all cash flow, denying Firm 2 the possibility to compensate for its inefficiency by an aggressive investment strategy. This strategy is less effective in competition with imperfect substitutes; the less so, the more differentiated the products. To summarize, myopic maximization leads to a different result precisely because firms cannot foresee that pressing "their advantage hard over disadvantaged firms by expanding their capital stock" (Nelson and Winter (1982, p. 311)) can win a market in the long run. Rational expectations leave no room for inefficiency as each firm always chooses the optimal strategy. Perhaps it is unrealistic to bestow the firm with perfect insight and foresight, bur rational expectations are, nonetheless, an important benchmark. Extensive literature on adaptive learning shows that even very naïve strategies (expectations) often converge to the rational expectations equilibrium in the long run (Dawid and Kopel (1998); Franke (1998); survey by Arifovic (2000)).

3.2 Complementary products

Competition ceases when all products are mutual complements. With $C_1,\ C_2 < 0$ conditions (19)-(20) are always satisfied and the duopoly becomes super-stable. Moreover, firms grow faster and attain output levels higher than would otherwise be possible with homogeneous or imperfectly substitutable products. The situation in which one firm expands while the other contracts that characterizes competition in imperfect substitutes does not arise with complementary products.

More interesting therefore are demand patterns that simultaneously account for the degree of substitutivity between some products and complementarity between others. While the endless variety of possible combinations defies meaningful theoretical classification, the necessity of tracking many firms suggests numerical simulation as the only practicable approach to solving models of such complexity.

3.3 Beyond duopoly

Despite the manifest weakness of evoking evolutionary dynamics for a population of two firms, many of the insights so gained carry over to markets with more than two firms. In the case of homogeneous products the model is readily generalized, leading to an N-firm analogue of the familiar knife-edge condition (11) (Appendix A.1).

Although generalizations are not easily obtained in the case of differentiated products, so far is certain: while a knife-edge equilibrium is the only kind of coexistence equilibrium with homogeneous products, differentiated products at least allow equilibria of the niche type. The size of the niche will depend on the characteristics of all firms which, together with consumer preferences, define the operating environment. Numerical simulations by the author (available upon request) confirm that equilibria of the niche type remain common for $N > 2$.

4 Summary

This paper shows that the stability of a duopoly profoundly depends on the underlying demand structure. In the case of homogeneous products the precondition for the coexistence of the two firms is knife-edge. Essentially this means that unless both firms are identical only one can survive in the long run. Monopoly is the likely outcome of competition in homogeneous products. Although product differentiation does not preclude monopoly, it greatly improves the chances for the stable coexistence of firms in the long run. The more differentiated the products, the more stable the duopoly.

Product differentiation allows firms to insulate their own market segment by reducing its sensitivity to competitive moves. This is shown on the example of investment strategy. While aggressive investment behavior inevitably increases competitive selection pressure, its effect diminishes as products become less substitutable. The same principle applies to other forms of strategic behavior, for example, to the choice of technology. Put in ecological terms, product differentiation creates a niche for the firm and is a factor that limits market concentration. It thus provides a rationale for the persistence in the observed heterogeneity of entrepreneurial forms and behaviors.

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A Appendix

A.1 Stable duopoly with homogeneous products

Let $\tilde{Q} = Q_* = Q^*$. After substituting equalities (11) into the system of growth equations

$$\dot{q}_1 = \frac{\lambda_1 a_1}{v_1}[h(Q) - h(\tilde{Q})]q_1 \; ; \tag{22}$$

$$\dot{q}_2 = \frac{\lambda_2 a_2}{v_2}[h(Q) - h(\tilde{Q})]q_2 \; , \tag{23}$$

observe that

$$\frac{\dot{q}_1}{q_1} \cdot \frac{v_1}{\lambda_1 a_1} = \frac{\dot{q}_2}{q_2} \cdot \frac{v_2}{\lambda_2 a_2} \; . \tag{24}$$

By integration

$$q_1 = C q_2^{\alpha} \quad \text{for} \quad C = \frac{q_1(0)}{q_2(0)^{\alpha}} \quad \text{and} \quad \alpha = \frac{\lambda_1 a_1 v_2}{\lambda_2 a_2 v_1} \; . \tag{25}$$

Evidently, outputs evolve according to $q_1(t) = C(q_2(t))^{\alpha}$. Since $Q(t) = q_1(t) + q_2(t)$ and $\lim_{t \to 0} Q(t) = \tilde{Q}$,

$$C(q_2(t))^{\alpha} + q_2(t) = \tilde{Q} + o_t(1) \; , \quad \text{where} \quad \lim_{t \to 0} o_t(1) = 0 \; . \tag{26}$$

Every limit point of $q_2(t)$ satisfies the equation $Cx^{\alpha} + x = \tilde{Q}$. Since the left-hand side is a strictly increasing function of x for $x > 0$, it must have a unique positive solution q_2^*. With $\lim_{t \to 0} q_2(t) = q_2^*$, the equilibrium market shares are given by $q_2^*/\tilde{Q}$ and $1 - q_2^*/\tilde{Q}$. Note that the equilibrium market shares depend non-linearly on all model's parameters and are positive for positive initial capital endowments.

In general, among $N > 2$ firms there will be $1 \leq k \leq N$ firms that fulfill condition (11). Since the remaining $N - k$ firms vanish, the equilibrium is determined by the dynamics of the extant firms only. For them we have

$$q_j = C_j q_1^{\alpha_j} \quad \text{for} \quad C_j = \frac{q_j(0)}{q_2(0)^{\alpha_j}} \quad \text{and} \quad \alpha = \frac{\lambda_j a_j v_1}{\lambda_1 a_1 v_j} \; , \quad \text{where} \quad j = 2, 3, \ldots, k \; . \tag{27}$$

The equilibrium market share of Firm 1 satisfies an N-firm analogue of equation (26)

$$q_1^* + C_2(q_1^*)^{\alpha_2} + \cdots + C_k(q_1^*)^{\alpha_k} \ . \tag{28}$$

Equilibrium market shares of the remaining $k-1$ firms can be found from equation (27).

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