

REGULARITY AND COINTEGRATION IN DEMAND SYSTEMS

Kurt Kratena
Michael Wüger

Austrian Institute of Economic Research
P.O. Box 91 , A-1103 Vienna, AUSTRIA
Tel.: +43 1 7982601 246
Fax: +43 1 7989386
e-mail: kratena@wsr.ac.at

Abstract: This paper deals with the regularity problem in the Almost Ideal Demand System (AIDS) in terms of the boundedness of the budget shares within the $[0,1]$ interval. The violation of ‘cointegration accounting’ can be seen just as another consequence of irregularity. The discussion of modifications in the PIGLOG cost functions leads to the proposition of a modified dynamic PIGLOG formulation, from which a system of budget shares can be derived, which ensures regularity and validity of ‘cointegration accounting’. A special feature of the resulting share equations is the divergence from the time path of the AIDS shares over time.

Key words: AIDS model, PIGLOG specification, cointegration

JEL classification: D11, D12, C30, C32

1. Introduction

Demand systems have become an important instrument of microeconomic research. Especially the empirical application of the Almost Ideal Demand System (AIDS) put forward by Deaton, Muellbauer (1980) has gained scope due to increasing use of multisectoral models for policy simulation and forecasting. The experience with these models often exhibits the importance of regularity for empirical application, especially at a very disaggregated level. The main empirical problem of regularity in AIDS is ‘that, under large changes in real incomes, budget shares can stray outside the $[0,1]$ interval’ (Rimmer, Powell (1996)). The work of Rimmer, Powell (1996) and of Almon (1998) can be seen as two recent examples, where the empirical problems with regularity of AIDS have led to the formulation of a completely new demand system. Cooper, McLaren (1992) also start in their work from the violation of regularity in AIDS and conclude to propose a modified PIGLOG class of preferences, from which a modified AIDS system (MAIDS) can be derived.

This study departs from the work of Cooper, McLaren (1992), who mention in their work as a side aspect of non - regularity, that ‘cointegration accounting’ is also not satisfied in the share equation system of AIDS, which is an important point in the application of dynamic AIDS. A consistent formulation of the cointegrating equation can be seen as the prerequisite for setting up a dynamic AIDS model. In this study the conditions of regularity and of ‘cointegration accounting’ shall be taken together as the starting point for the derivation of empirically oriented PIGLOG preferences specifications. It is shown, that the MAIDS proposed by Cooper, McLaren (1992) satisfies these two conditions. That leads to another

alternative dynamic PIGLOG preferences formulation, where the budget shares equation system has the property that the shares are not only a positive function of real expenditure but also a negative function of the difference between actual real expenditure and the expenditure level at the starting point $t = 0$. The time path of the budget share for a luxury in this system is very close to a logistic function and the system also satisfies regularity and cointegration accounting. In the share equation system AIDS can be seen then as a special case, which is valid at $t=0$. An empirical comparison of this dynamic MAIDS with AIDS and with MAIDS of Cooper, McLaren (1992) shows rather different parameter values for the three systems.

2. Regularity and Cointegration in AIDS

The starting point of the analysis is the PIGLOG - specification of the cost function in AIDS:

$$(1) \ln C(u,p) = (1 - u) \ln (a(p)) + u \ln (b(p))$$

where $a(p)$, $b(p)$ are positive and homogenous of degree one functions in a vector of prices p and u is a given level of utility. This general cost function is transformed into the AIDS model by specifying $a(p)$ as a Translog function and $\ln b(p)$ as a Cobb - Douglas function (Deaton, Muellbauer (1980)):

$$(2) \ln a(p) = a_0 + \sum_k a_k \ln p_k + 0,5 \sum_{kj} \gamma_{kj}^* \ln p_k \ln p_j$$

$$(3) \ln b(p) = \ln a(p) + \beta_0 \prod_k p_k^{\beta_k}$$

Cooper, McLaren (1992) write (1) as:

$$(4) \ln C(u,p) = \ln P_1 + uP_2$$

with

$$P_1 = a \text{ and } P_2 = \ln(b/a)$$

To this cost function an indirect utility function exists, which can be derived by rearranging

(1):

$$(5) U(c, p) = [\ln (c / P_1)] (1 / P_2)$$

Using the notation of Cooper, McLaren (1992) for

$$\epsilon_{ki} = \delta \ln P_k / \delta \ln p_i \quad k = 1, 2 \quad \text{and} \quad i, j = 1 \dots \dots n \quad (\text{for } n \text{ goods})$$

$$R = \ln (c / P_1) \quad \text{real expenditure} = (c / P_1)$$

and Shephard's Lemma to derive the budget shares $w_i = p_i q_i / c$:

$$(6) \quad w_i = \frac{\delta \ln (C(u, p))}{\delta \ln p_i}$$

one can use (5) to derive Cooper, McLaren's (1992) formulation for the budget share system:

$$(7) \quad w_i = \epsilon_{1i} + \epsilon_{2i} R$$

with the restrictions $\sum_i \epsilon_{1i} = 1$ and $\sum_i \epsilon_{2i} = 0$ following from $P_1 = HD1$ (homogenous of degree 1) and $P_2 = HD0$ (homogenous of degree 0).

From this formulation it becomes obvious that the bounded variable w_i must at some point stray outside the $[0, 1]$ interval, if R increases.

Cooper, McLaren (1992) also mention the aspect that (7) does not satisfy for all possible values of R the rules of conintegration accounting, as the bounded variable w_i is a function of

the non - bounded variable R. To make this point explicit, let us assume that the time path of R can be described by a random walk with drift process:

$$(8) R_t = R_0 + \beta_0 t + \sum_{n=1}^t \eta_n$$

$$(9) \Delta R_t = \beta_0 + \eta_t$$

The share equations (7) can be written in a dynamic formulation as:

$$(10) w_{it} = w_{i0} + \varepsilon_{2i} \beta_0 t + \sum_{n=1}^t \eta_n$$

To set up the procedure of finding a cointegrating equation between w_i and R one would start with stationarity tests (Engle, Granger (1987)). A test on stationarity of R would yield that R is I(1) and ΔR is I(0), i.e. that R is difference stationary. For some sample range one might also end up with the result that the budget shares w_i are I(1). Running a regression of type (7) one would therefore get the stationary linear combination of w_i and R :

$$(11) v_t = w_{it} - \varepsilon_{1i} - \varepsilon_{2i} R_t$$

Actually this is often done, before setting up a dynamic AIDS model. It must be noted however, that this cointegrating relationship only exists for a certain sample range or - in economic terms - for a certain region of the expenditure-price space. As w_i is bounded by the

[0,1] interval, v_t cannot be stationary for the whole expenditure-price space, or to put it the other way, a forecast based on (11) will violate regularity in terms of the boundedness of w_i .

The properties of AIDS which lead to violation of regularity can be seen in the derivatives of (7). The response of the budget share to growth in expenditure is $\delta w_i / \delta \ln c = \epsilon_{2i}$. A constant response of the budget share must at some point of the price-expenditure space lead to irregularity. The same result can be obtained by looking at the time path of the budget shares, which is driven by the linear deterministic change β_0 , so that $\delta w_{it} / \delta t$ can be derived from (7) and (8) with:

$$(12) \quad \frac{\delta w_{it}}{\delta t} = \epsilon_{2i} \beta_0$$

From (12) the link between the violation of regularity and of cointegration accounting becomes also obvious. At some point t the budget shares w_{it} must lie outside the [0,1] interval, because due to the deterministic component of the random walk model $\delta w_{it} / \delta t$ is a constant and $\delta^2 w_{it} / \delta t^2 = 0$. The expenditure elasticity, $\delta \ln q_i / \delta \ln c$, can be derived by:

$$(13) \quad E_i = 1 + (\epsilon_{2i} / w_i)$$

3. Cointegration in MAIDS (modified AIDS)

Cooper, McLaren (1992) propose an alternative PIGLOG formulation of preferences, which allows the derivation of a system with bounded budget shares given rising expenditure. Their solution parts from the alternative cost function (with the same notation as in (4)) :

$$(14) \ln C(u,p) = \ln P_1 + uP_2 / [C(u,p)]^\phi$$

where P_1 is a homogenous of degree one function in the vector of prices p and P_2 is a homogenous of degree ϕ function in p .

The indirect utility function for this PIGLOG specification is:

$$(15) U(c,p) = [\ln (c/ P_1)] (c^\phi / P_2)$$

The PIGLOG specification of AIDS described in (4) and (5) can be seen as a special case of (14) and (15) when $\phi = 0$. Cooper, McLaren (1992) derive the restriction $0 \leq \phi \leq 1$ from a discussion of regularity of the indirect utility function. We will limit ourselves in this study to discuss regularity with directly analysing the budget share equations of a demand system.

Using again the notation

$$\varepsilon_{1i} = \delta \ln P_1 / \delta \ln p_i, \quad \varepsilon_{2i} = \delta \ln P_2 / \delta \ln p_i, \quad R = \ln (c/P_1)$$

one derives the solution of Cooper, McLaren (1992) for the share system by applying

Shephard's Lemma and Roy's identity: ¹

$$(16) w_{it} = (\varepsilon_{1i} + \varepsilon_{2i} R_t) / (1 + \phi R_t)$$

with the restrictions $\sum_i \varepsilon_{1i} = 1$ and $\sum_i \varepsilon_{2i} = \phi$ following from $P_1 = \text{HD1}$ (homogenous of degree 1) and $P_2 = \text{HD } \phi$ (homogenous of degree ϕ).

In the region $c \in P_1$ the restrictions $\varepsilon_{1i} \geq 0$ and $\varepsilon_{2i} \geq 0$ suffice to guarantee $0 < w_i < 1$.

It becomes also obvious that w_i is not only a positive function of the I(1) variable R determined by ε_{2i} as in (7), but also a negative function of R determined by the value of ϕ and the response of the budget share to growth in expenditure, $\delta w_i / \delta \ln c$, is not a constant:

$$(17) \quad \frac{\delta w_{it}}{\delta \ln c_t} = \frac{\varepsilon_{2i} - w_{it} \phi}{1 + \phi R_t}$$

Equation (17) shows the negative impact of the expenditure level and of the value of the share on the response of shares to growth in expenditure. By defining the variable $Z = \phi R_t / (1 + \phi R_t)$ Cooper, McLaren (1992) end up with an expression with bounded variables on both sides, so that the conditions of cointegration accounting are fulfilled:

$$(18) \quad w_{it} = \varepsilon_{1i} (1 - Z) + (\varepsilon_{2i} / \phi) Z$$

The insight, that the budget shares of MAIDS are bounded can also be derived from the time path of w_i compared to the time path in AIDS:

$$(19) \quad \frac{\delta w_{it}}{\delta \ln c_t} = \frac{\varepsilon_{2i} \beta_0 - w_{it} \phi \beta_0}{\beta_0}$$

¹ Roy's Identity allows to work only with the indirect utility function and to get the derivative of the cost function through: $(\delta C / \delta p_i) = (- \delta U / \delta p_i) / (\delta U / \delta C)$

$$\delta t \quad 1 + \phi R_0 + \phi \beta_0 t$$

where $\delta w_{it}/\delta t$ is not a constant as in (12) and $\delta^2 w_{it}/\delta^2 t$ has the opposite sign of $\delta w_{it}/\delta t$. The time path of the budget share in AIDS (12) can again be derived as a special case when $\phi = 0$.

One can see from (19) that the time path of w_{it} in MAIDS has the following features of a logistic curve : (i) it is a negative function of the actual level of w_{it} and (ii) it is a negative function of t . The expenditure elasticity in MAIDS becomes:

$$(20) E_i = [1/(1 + \phi R) (\epsilon_{2i}/w_i - \epsilon_{1i} \phi /w_i)] [1 - Z]$$

and reduces to the expenditure elasticity of AIDS $(1 + (\epsilon_{2i}/w_i))$ in the limiting case $\phi = 0$, as $Z = 0$, when $\phi = 0$.

The work of Cooper,McLaren (1992) further shows, that the share equations of MAIDS (16) are rather different from the AIDS equations for the whole expenditure-price space except for the limiting case $\phi = 0$. This is also the result of the empirical work of Cooper,McLaren (1992), where they end up with remarkable differences in parameter values for ϵ_{1i} and ϵ_{2i} between AIDS and MAIDS. This difference reflects the introduction of parameter ϕ to guarantee regularity and validity of cointegration accounting for the share equations.

4. A Dynamic PIGLOG Specification

We pick up the idea of Cooper, McLaren (1992) that an empirically oriented demand system can be a modification of the AIDS specification. In a recent paper Cooper, McLaren (1996) show another general PIGLOG specification, from which AIDS and the linear expenditure system (LES) can be derived as special cases and which also ensures regularity.

We want to find a modification of AIDS, where (i) the share equations guarantee regularity in the sense of the boundedness of the shares (ii) the static model can be seen as a consistent cointegrating relationship and (iii) the PIGLOG preferences formulation as well as the derived share equations of AIDS can be seen as a special case.

For this purpose we propose the following modifications of the indirect utility function:

$$(21) U_t(c_t, p_t) = [\ln(c_t / P_{1,t})] ((c_t / c^*)^p / P_{2,t})$$

$$(22) U_t(c_t, p_t) = [\ln(c_t / P_{1,t})] ((c_t - c^*)^p / P_{2,t})$$

where c^* may be seen as a reference value of expenditure similar to the minimum income level in the linear expenditure system. In (21) and (22) we choose a dynamic formulation taking into account that R is a time series following a random walk with drift process and we are interested to analyse regularity and validity of cointegration accounting simultaneously.

The PIGLOG specification of AIDS described in (5) can as in MAIDS be seen as a special case of (21) for the trivial case that $\rho = 0$. The share equations can again be derived by applying Shephard's Lemma and Roy's identity. For this purpose we define the reference value c^* as a certain level of expenditure within the sample range. We propose that c^* having the character of a minimum value should be specified so that $\ln c^* = R_0$ and $R_0 = \ln(c_0 / P_{1,0})$.

We get the share equations as:

$$(23) \quad w_{it} = (\varepsilon_{1i} + \varepsilon_{2i} R_t) / (1 + \rho R_t - \rho R_0)$$

where as in (16) $\sum_i \varepsilon_{1i} = 1$, but the restriction for ε_{2i} becomes time dependent now:

$\sum_i \varepsilon_{2i} = \rho (1 - R_0/R_t)$ and $P_1 = \text{HD}1$ (homogenous of degree 1) and $P_2 = \text{HD} \rho$ (homogenous of degree ρ). As in MAIDS in the region $c = P_1$ the restrictions $\varepsilon_{1i} \geq 0$ and $\varepsilon_{2i} \geq 0$ are sufficient to ensure boundedness of the shares ($0 \leq w_i \leq 1$).

As already mentioned we see from the modified PIGLOG formulation, that the PIGLOG formulation of AIDS nests (21) when $\rho = 0$, which can be seen as the general limiting case.

But there is another particular limiting case in the share equations, when AIDS nests this dynamic MAIDS formulation, namely at $R_t = R_0$. So it is a special feature of the derived share equations that they begin to diverge from the time path of the AIDS shares from $t = 0$ on. The difference $(R_t - R_0)$ plays an important role in the determination of w_{it} . From (8) we know that R follows a random walk plus drift process, so that this difference is a function of t due to the deterministic trend component β_0 .

$$(24) \quad R_t - R_0 = \beta_0 t + \sum_{n=1}^t \eta_n$$

There is a direct feed back therefore in (23) from the deterministic trend component β_0 on the shares as expenditure R_t grows. This feed back depends also on ρ and ensures boundedness and the validity of cointegration accounting. The analysis of boundedness of the shares can as in MAIDS also be carried out by defining a bounded variable using R_t , which in this case is $Y = (\rho R_t - \rho R_0) / (1 + \rho R_t - \rho R_0)$. That allows to express the share equations with:

$$(25) w_{it} = \varepsilon_{1i} (1 - Y) + (\varepsilon_{2i}/\rho) [Y(1 - \Phi) + \Phi]$$

where the constant $\Phi = \rho R_0$ and where the same restrictions as before for $\sum_i \varepsilon_{1i}$ and for $\sum_i \varepsilon_{2i}$ guarantee additivity of the share system.

The response to growth in expenditure and the time path of the shares are given with:

$$(26) \quad \frac{\delta w_{it}}{\delta \ln c} = \frac{\varepsilon_{2i} - \rho w_{it}}{1 + \rho R_t - \rho R_0} = \frac{\varepsilon_{2i} - \rho w_{it}}{1 + \rho \beta_0 t}$$

$$(27) \quad \frac{\delta w_{it}}{\delta t} = \frac{\varepsilon_{2i} \beta_0 - \rho w_{it} \beta_0}{1 + \rho \beta_0 t}$$

That means that if we look at a dataset for a luxury, which starts with very small values of the share, the time path at the beginning will be very similar to the one implied in AIDS (equation (12)). The most important feature of the resulting share equations of the proposed dynamic PIGLOG formulation therefore is the process of divergence from the AIDS shares as t increases, while AIDS is the limiting case at $t = 0$. This is achieved at the expense that the restrictions for additivity concerning ε_{2i} become time varying.

The expenditure elasticity, E_i , which can be derived from this dynamic MAIDS is similar to MAIDS showing the same properties in terms of being a negative function of the bounded variable Y and being the same as in AIDS $(1 + (\varepsilon_{2i}/w_i))$ when $\rho = 0$:

$$(28) E_{it} = 1 + [(1/(1 + \rho R_t - \Phi)) (\epsilon_{2i}/w_{it} - \epsilon_{1i} \rho / w_{it})] [1 - Y]$$

5. An Empirical Comparison between AIDS, MAIDS and Dynamic MAIDS

Demand systems of the AIDS type have been applied in several fields of demand analysis.

The problems of regularity become especially relevant in the two goods case, where a rising

share with positive response to total expenditure is combined with a decreasing share with negative response to total expenditure. A typical example of such a demand system is the import demand model of Anderton, Pesaran, Wren-Lewis (1992). The empirical example chosen in this paper is an import demand model for total imports of Austria, where total demand is defined as the sum of GDP and imports and the share system describes the allocation between imported and domestic demand.

Following the lines of Cooper, McLaren (1992) the share equation systems derived above from the PIGLOG specification can be seen as the description of the individual behaviour (e.g. of a household), whereas the data observable from statistics (in the case of the import model from National Accounts) usually are macro - data. The link between these two levels is given by the rules of aggregation. As is well known, there is no aggregation problem in the case of identical preferences and an equal distribution of income. Looking at the share equations as describing the micro level and writing for „micro Z“ : Z^* and for „micro Y“ : Y^* we may for convenience reproduce the three different systems outlined above. If we say that the micro level describes households behaviour, we can define an aggregation rule over households (h) for Z and Y:

$$(7) \quad w_i = \varepsilon_{1i} + \varepsilon_{2i} R \quad \text{AIDS}$$

$$(18a) \quad w_{it} = \varepsilon_{1i} (1 - Z^*) + (\varepsilon_{2i}/\phi) Z^* \quad \text{MAIDS}$$

$$(25a) \quad w_{it} = \varepsilon_{1i} (1 - Y^*) + (\varepsilon_{2i}/\rho) [Y^*(1 - \Phi) + \Phi] \quad \text{Dynamic MAIDS}$$

$$(29) Z^* = \frac{\sum_h c^h Z^h}{\sum_h c^h}, \quad Y^* = \frac{\sum_h c^h Y^h}{\sum_h c^h}$$

This allows to describe the macro share systems for MAIDS and dynamic MAIDS in terms of macro Z and Y and of the difference between the macro and the micro level ($Z - Z^*$) and ($Y - Y^*$):

Macro - MAIDS

$$(30) w_{it} = \varepsilon_{1i} (1 - Z) + (\varepsilon_{2i}/\phi) Z + (\varepsilon_{1i} - \varepsilon_{2i}/\phi) (Z - Z^*)$$

Dynamic Macro - MAIDS

$$(31) w_{it} = \varepsilon_{1i} (1 - Y) + (\varepsilon_{2i}/\rho) [Y(1 - \Phi) + \Phi] + (\varepsilon_{1i} - \varepsilon_{2i}/\phi(1 - \Phi)) (Y - Y^*)$$

The difference ($Z - Z^*$) and ($Y - Y^*$) is driven by variables, which represent differentiation of preferences, in the case of consumption the most important variable may be a measure of the distribution of real spending power. A further step to the empirical specification is therefore to approximate the terms $(\varepsilon_{1i} - \varepsilon_{2i}/\phi) (Z - Z^*)$ and $(\varepsilon_{1i} - \varepsilon_{2i}/\phi(1 - \Phi)) (Y - Y^*)$ by

$$(32) (\varepsilon_{1i} - \varepsilon_{2i}/\phi) (Z - Z^*) = \phi \mu_i x$$

$$(33) (\varepsilon_{1i} - \varepsilon_{2i}/\phi(1 - \Phi)) (Y - Y^*) = \rho \mu_i x$$

where x is a vector of explanatory variables for the differences ($Z - Z^*$) and ($Y - Y^*$) and μ_i are parameters with $\sum_i \mu_i = 0$.

The final step to derive the empirical specification of the share equation systems consists in using the definitions for ε_{1i} and ε_{2i} given the Translog function for P_1 and the Cobb - Douglas function for P_2 :

$$(34) \ln P_1 = \alpha_0 + \sum_i \alpha_i \ln p_i + 0,5 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$$

$$(35) \ln P_2 = \sum_i \beta_i \ln p_i$$

where

$$(36) \varepsilon_{1i} = \delta \ln P_1 / \delta \ln p_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j \quad ; \quad \varepsilon_{2i} = \delta \ln P_2 / \delta \ln p_i = \beta_i \quad ; \quad R = \ln (c/P_1)$$

The share equation systems of the three models (37) to (39) have been applied to an import demand system for Austria at the aggregate level with quarterly data from 1964:1 to 1994:4. The import share of the Austrian economy is steadily rising and one factor behind this process is the continuous opening up of the small open economy Austria. A variable for x had to be chosen to measure this opening up process, which works at the aggregate level. It was decided to take the aggregate export share (exports/(exports + GDP)) of the Austrian economy for this sake.

$$(37) w_{it} = \varepsilon_{1i} + \beta_i R_t + \mu_i x \quad \text{AIDS}$$

$$(38) w_{it} = (\varepsilon_{1i} + \beta_i R_t) / (1 + \phi R_t) + \phi \mu_i x \quad \text{MAIDS}$$

$$(39) w_{it} = (\varepsilon_{1i} + \beta_i R_t) / (1 + \rho R_t - \rho R_0) + \rho \mu_i x \quad \text{Dynamic MAIDS}$$

All estimations have been carried out by system estimation (SURE) using the explicit formulation for the price index $\ln P_1$ and not the approximation of the Stone - index ($\ln P_1^* = \sum_i w_i \ln p_i$) which involves the estimation constrained to the symmetry restriction. The homogeneity restriction has not been imposed in neither of the three systems. The adding up restrictions in AIDS are fulfilled automatically, the special adding up restriction in MAIDS, $\sum_i \beta_i = \phi$ and in dynamic MAIDS $\sum_i \beta_i = \rho (1 - R_0 / R_t)$ have been imposed by substituting parameters. In the case of dynamic MAIDS the values of R_0 and R_t within the terms introduced to impose the adding up restriction have been derived by approximating $\ln P_1$ by the Stone - index. Furthermore in dynamic MAIDS in order to reduce complexity the term R_0 / R_t in the additivity restriction has been introduced by inserting the sample mean of R_t . That implies the validity of the additivity restriction at the average of the sample.

The results first of all show the applicability of the MAIDS specification for this demand system, as the parameters ϕ and ρ are based on significant values for the parameters β_i . The most important result is the difference in the parameters between MAIDS, dynamic MAIDS and AIDS. That holds for the response to real expenditure (in this case total demand) of the shares measured by the parameter β_i as well as for the parameters γ_{ij} . In this aspect the results of Cooper, McLaren (1992) are confirmed, that the parameters are rather different in AIDS and in MAIDS. This has consequences for the expenditure elasticity, which is rather different for the same commodity in the two systems. The results for the R^2 and the DW - statistics

show that both MAIDS and dynamic MAIDS may be seen as a valid cointegrating equation and as the starting point of a dynamic specification.

*Table 1: Empirical comparison of AIDS, MAIDS and Dynamic MAIDS
for an import demand system*

	AIDS		MAIDS		Dynamic MAIDS	
	imports	domestic	imports	domestic	imports	domestic
	(M)	(D)	(M)	(D)	(M)	(D)
$\alpha(i)$	-0,683	1,642	-1,007	1,585	0,286	0,679
$\beta(i)$	0,067	-0,063	0,103	-0,062	0,033	0,009
$\gamma(iM)$	0,041	-0,031	0,082	-0,077	0,396	-0,119
$\gamma(iD)$	-0,031	0,023	-0,077	0,085	-0,119	0,035
$u(i)$	0,343	-0,352	5,578	-5,644	0,615	-0,617
$\phi(i)$	--		0,042			
$\rho(i)$	--				0,463	
R^2	0,875		0,887		0,886	
DW	1,563		1,563		1,053	

References

Almon, C., A Perhaps Adequate Demand System with Application to France, Italy, Spain and the USA, Department of Economics, University of Maryland, 1998

Anderton, B., Pesaran, B., Wren-Lewis, S., Imports, Output and the Demand for Manufactures, Oxford Economic Papers, 1992, (44),

Cooper, R. J., McLaren, K. R., An empirically oriented demand system with improved regularity properties, Canadian Journal of Economics, 1992, (25), 652 - 667

Cooper, R. J., McLaren, K. R., A system of demand equations satisfying effectively global regularity conditions, Review of Economics and Statistics, 1996, 359 - 364

Deaton, A., Muellbauer, J., An Almost Ideal Demand System, American Economic Review, 1980, 312 - 326

Engle, R.F., Granger, C.W.J. , Cointegration and Error Correction: Representation, Estimation and Testing, Econometrica, 1987, 251 - 276

Rimmer, M., T., Powell, A., A., An Implicitly Additive Demand System, Applied Economics, 1996 (28), 1613 - 1622