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Risk-Averse Monopolist with Aspiration

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Abstract

This paper analyzes optimizing decisions of a monopolist under uncertainty. The aspiration model directly accounts for asymmetric risk preferences with respect to downside risk. The optimal output (price) of a risk-averse monopolist facing marginal cost uncertainty will not exceed that of his risk-neutral counterpart, and will be lower (higher) for realistic aspiration levels. This result is consistent with studies conducted in the traditional expected utility framework

Key Words: aspiration, downside risk, monopoly, risk-aversion

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The behavior of a firm facing uncertainty is a central issue in industrial economics. The existing literature has shown that optimizing decisions under uncertainty depend on the nature of uncertainty and the assumptions concerning the risk preference of the firm. Specifically, it seems important to distinguish between uncertainty in costs and uncertainty in demand, and whether the firm is risk-neutral or behaves as if it were risk-averse.[1] It is important to ask how robust are the results of this literature to the choice of the model of risk-taking behavior.

Previous studies of effects of risk-aversion on the firm's behavior have been conducted in the traditional expected utility framework. The salient feature of the expected utility theory is that concavity of the expected utility function is equivalent to risk aversion, whereby the more concave the expected utility function, the more risk averse the firm. At the implementation level, the specification of the utility function often requires the use of the variance as a measure of risk. The main criticism of the expected utility theory is that it treats downside and upside risks symmetrically, whereas risk is typically perceived as an asymmetric phenomenon, see prospect theory of Kahneman and Tversky (1979). According to prospect theory, a utility function is typically concave for gains, convex for losses, and has a kink at the aspired level of utility. An alternative consistent with prospect theory is the use of downside risk measures, where risk is associated with failure to attain a level of aspiration. The attractiveness of downside risk measures over the mean-variance approach is well established in finance, and is also supported by experimental evidence and behavioral psychology.[2] Downside risk measures can be reconciled with an expected utility framework by the use of discontinuous utility functions, see Fishburn (1977).

This paper analyzes optimizing decisions of a monopolist subject to uncertainty in the level of marginal costs using an alternative, and, perhaps, more realistic model of risk aversion. The model directly accounts for asymmetric risk preferences with respect to downside risk. In the deterministic case, a monopolist facing a continuously twice-differentiable, strictly decreasing inverse demand function h and a linear cost function seeks to maximize the profit $\Pi_m(q) = h(q)q - mq - c$, where $q, m, c > 0$ are, respectively, the

[1] Early work on monopoly includes Baron (1971) and Leland (1972). A recent article by Asplund (2002) studies oligopolistic competition with risk-averse firms and includes monopoly as a special case. This article also contains references on related studies in the framework of perfect and oligopolistic competition.

[2] See Siegmann and Lucas (2001) for an example in the finance literature, and Kahneman and Tversky (1979) for early examples of laboratory studies.

quantity sold, constant marginal and fixed costs of production. If one makes the usual assumption that $\Pi''_m(q) < 0$, then $\Pi_m(q)$ becomes strictly concave in q, and the maximizer q^*_m is unique. Let us introduce uncertainty in the level of marginal costs. Specifically, let m be a random variable having an absolutely continuous probability distribution function F with support $(0, \infty)$, a probability density function $f = F'$, and a finite expected value $\mathsf{E}m = \mu$.

Denote the utility function of a risk-neutral monopolist by $U_1(q)$ and that of a risk-averse by $U_2(q)$. The standard optimization problem of a risk-neutral monopolist is

$$\max_q U_1(q) = \mathsf{E}\Pi_m(q) \; . \tag{1}$$

Specification of the following aspiration model has been adopted from Siegmann and Lucas (2001). The optimization problem of a risk-averse monopolist can be written as

$$\max_q U_2(q) = \mathsf{E}\Pi_m(q) - \lambda \mathsf{E}[B - \Pi_m(q)]^+ \; , \tag{2}$$

where $[\cdot]^+$ is a short-hand notation for $\max[\cdot, 0]$. The first term is the monopolist's expected profit $\mathsf{E}\Pi_m(q) = \Pi_\mu(q)$. The second term is the expected profitability shortfall measured against a level of aspiration B, which is exogenous to the model, but is assumed to exceed the fixed costs, i.e. $B > c$. The risk aversion parameter λ determines the monopolist's preference for risk taking, or the trade-off between the expected profit and the downside risk. The case $\lambda = 0$ corresponds to a risk-neutral monopolist, whereas $\lambda > 0$ to a risk-averse.

If $\Pi_m(q)$ is strictly concave in q, as has been assumed, then so are $U_1(q)$ and $U_2(q)$. The concavity of $U_1(q)$ readily follows from $U_1(q) = \Pi_\mu(q)$. Given that $\Pi_m(q)$ is strictly concave, $B - \Pi_m(q)$ is strictly convex, $[B - \Pi_m(q)]^+$ is convex and so is $\mathsf{E}[B - \Pi_m(q)]^+$. Finally, $-\lambda \mathsf{E}[B - \Pi_m(q)]^+$ is concave, as $\lambda > 0$. It follows that $U_2(q)$, as a sum of a strictly concave function and a concave function, is strictly concave.

Let q^*_μ, q^* be, respectively, the optimal quantity set by a risk-neutral and a risk-averse monopolist. Given the strict concavity of $U_1(q)$ and $U_2(q)$, the optimal outputs can be compared by asserting the sign of $U_2'(q_\mu^*)$. One of the following three cases may occur:

$$\frac{d}{dq}\mathsf{E}[B-\Pi_m(q_\mu^*)]^+ < 0 \quad \Leftrightarrow \quad U_2'(q_\mu^*) > U_1'(q_\mu^*) = 0 \quad \Leftrightarrow \quad q_\mu^* < q^* \ ; \tag{3}$$

$$\frac{d}{dq}\mathsf{E}[B-\Pi_m(q_\mu^*)]^+ = 0 \quad \Leftrightarrow \quad U_2'(q_\mu^*) = U_1'(q_\mu^*) = 0 \quad \Leftrightarrow \quad q_\mu^* = q^* \ ; \tag{4}$$

$$\frac{d}{dq}\mathsf{E}[B-\Pi_m(q_\mu^*)]^+ > 0 \quad \Leftrightarrow \quad U_2'(q_\mu^*) < U_1'(q_\mu^*) = 0 \quad \Leftrightarrow \quad q_\mu^* > q^* \ . \tag{5}$$

Taking the derivative yields

$$D(q) \equiv \frac{d}{dq}\mathsf{E}[B-\Pi_m(q_\mu^*)]^+ = \int_{\bar{m}(q)}^{\infty} [m - h'(q)q - h(q)]f(m)dm \ , \tag{6}$$

where $\bar{m}(q) = h(q) - B/q$ (proof in Appendix). The derivative can now be evaluated at q_μ^* by substituting the first order condition for a risk-neutral monopolist, $h'(q_\mu^*)q_\mu^* + h(q_\mu^*) - \mu = 0$,

$$D(q_\mu^*) = \int_{\bar{m}(q_\mu^*)}^{\infty} [m-\mu]f(m)dm \ , \tag{7}$$

where $\bar{m}(q_\mu^*) = \mu - h'(q_\mu^*)q_\mu^* - B/q_\mu^*$. It can be shown that (7) is non-negative by considering the function $J(t) = \int_t^\infty [m-\mu]f(m)dm$. It is strictly increasing for $t<\mu$, and strictly decreasing for $t>\mu$, as $J'(t) = (\mu - t)f(t)$. Since, in addition, $J(0) = 0$ and $\lim_{t\to\infty} J(t) = 0$, it follows that $J(t) > 0$ for $t > 0$. Hence,

$$D(q_\mu^*) > 0, \quad q_\mu^* > q^* \qquad \text{for} \qquad h'(q_\mu^*)q_\mu^* + \frac{B}{q_\mu^*} < \mu \ ; \tag{8}$$

$$D(q_\mu^*) = 0, \quad q_\mu^* = q^* \qquad \text{for} \qquad h'(q_\mu^*)q_\mu^* + \frac{B}{q_\mu^*} \geq \mu \ . \tag{9}$$

The results are consistent with the existing literature and can be summarized as follows. First, the optimal output of a risk-averse monopolist facing marginal cost uncertainty will not exceed that of his risk-neutral counterpart, and will be lower for realistic aspiration levels. Indeed, given the non-negative support of m, the integral in (7) vanishes for sufficiently large values of B. Although, in this case the quantity chosen by a risk-averse and a risk-neutral monopolist coincide, the level of utility attained by a risk-averse monopolist is lower because of the penalty of not attaining the aspiration level set. The interpretation is that unattainable standards are irrelevant for decision-making, but adopting them reduces utility. Under the standard assumption of a strictly decreasing inverse demand function, the opposite is true for the optimal price. A risk-averse monopolist facing uncertainty in the level of marginal costs will thus be indifferent between quantity-setting or price-setting behavior. Second, the present analysis shows the optimal quantity and price of a risk-averse monopolist is not independent of the level of fixed costs, as is the case with a risk-neutral monopolist. Both results confirm those obtained in the expected utility framework by Asplund (2002). The present conclusions were obtained under the assumption of a linear cost function, but are otherwise quite general. In particular, no specific assumptions were made about the probability distribution function and the existence of moments higher that the first, such as the variance.

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A Appendix

Derivation of (6). Using the Leibnitz's Rule for differentiation of integrals

$$\begin{aligned}
\frac{d}{dq}\mathsf{E}[B-\Pi_m(q)]^+ &= \left\{\int\limits_{\bar{m}(q)}^{\infty}[B-h(q)q+mq]f(m)dm\right\}' = \\
&= \left\{\int\limits_{\bar{m}(q)}^{\infty}[B-h(q)q]f(m)dm + q\int\limits_{\bar{m}(q)}^{\infty}mf(m)dm\right\}' = \\
&= -\int\limits_{\bar{m}(q)}^{\infty}[h'(q)q+h(q)]f(m)dm - \underbrace{[B-h(q)q]f(\bar{m}(q))\bar{m}'(q)}_{1} + \\
&+ \int\limits_{\bar{m}(q)}^{\infty}mf(m)dm + \underbrace{q\bar{m}(q)f(\bar{m}(q))\bar{m}'(q)}_{2} = \\
&= \int\limits_{\bar{m}(q)}^{\infty}[m-h'(q)q-h(q)]f(m)dm\ ,
\end{aligned}$$

for $\bar{m}(q) = h(q) - \dfrac{B}{q}$, whence the terms 1 and 2 cancel for $q = q^*_\mu$.

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